Outline of superselectionrule on Algebraic Quantum Field Theory : Advanced

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Abstract

On algebraic quantum field theory, I will focus on the sector theory, the representation theory of operator algebras (von Neumann algebras) related to it, and the relationship with the theory of irreducible decomposition.

Keywords: sector theory, vacuum representation, factor, observation problem.

1 General issues of sector theory

A sector is, in the general framework of algebraic quantum field theory, " (the same class) superpositionable subspace sorted by a superselection rule (superselective charge)" [9]. Therefore, no superposition occurs between (vectors or representations of) two different sectors. In terms of the algebra of observables, sector can be described as the decomposition of the observable algebra (self-adjoint representation) into factor representations (irreducible decomposition representations)[1]. Sectors are generally classified by discrete superselective charges (projections), so situations in which continuous superselective charges appear are not often assumed. However, from the standpoint of broadly understanding the order parameter, which is a macro-indicator used in thermodynamics and other fields, the problem of observation becomes clearer. Here, we will start with the general part.

[1]As will be discussed later, in the case of type I factor, is not a problem because the representation of factor has the same meaning as the irreducible representation. However, introductory texts, such as H.Araki's are written from the point of view of von Neumann algebras with sectors of type I, which is why I chose irreducible decomposition here. From I.Ojima's point of view, or from a broader non-I-type point of view, it should be called "factorization", but here, in the sense of showing the problems of irreducible decomposition, the irreducible decomposition is also included in this paper. I followed a formula "from quantum mechanics with finite degrees of freedom to quantum field theory." I will return to this point at the end of this paper. One more thing to add here is that when we take the bounded linear operator $\mathcal{L}(\mathcal{H}) = (\cup\mathcal{M}(\mathcal{O}))''$ as the global operator in the vacuum representation, $\mathbf{C}1 = (\cup\mathcal{M}(\mathcal{O}))'$ is the meaning of the irreducible representation, which is the same as the uniqueness of the vacuum as described later, but This is also related to the fact that the world of type I von Neumann algebras actually captures the internal structure of the infinite quantum world largely (roughly). Although it cannot be arranged in this note, the paradox concerning the excitation of RQFT that E. Fermi once presented (E.Fermi, Quantum Theory of Radiation: in *J, Math Phys 4*, 87-132, 1932) is may be also caused.

1.1 Summary from the text of R.Haag and H.Araki

Here, the texts of [9] and [15] are mainly organized. The superselection rule assumes some charges that the excited state has as general charges as a quantity that distinguishes different localized excitations from the vacuum, and calls it notions. Since it is a localized excitation, charges in general electromagnetism are not localized according to Gauss' law. In general, the baryon number is often assumed.

[Vacuum sector]

Before considering the excited state, let us consider how to treat the "vacuum state".

In quantum field theory, the vacuum plays an important role with respect to "spontaneous symmetry breaking". In that case, how to select a vacuum becomes a problem. It is not well understood whether the vacuum is absolutely unique, or whether degeneracy-like states are common and there is no meaning in choosing a unique vacuum. There is no problem in assuming global uniqueness when dealing with free fields, but vacuum polarization occurs as soon as we assume interacting fields. A degenerate state can be seen as a case where unique vacancies are superimposed, but since interactions are always occurring in the real world, it is also possible to think that the degenerate vacancies are decomposed into individual vacancies.

The (degenerate) vacuum disperses in the limit where the volume of space is infinite. all of them are orthogonal even if a local operator is applied to them [4][5]. Therefore, globally the superposition of the vacuums dissolves, and the mutual vacuums become non-equivalent due to the orthogonality of the vacuums. In other words, it can be interpreted that the vacuum sector appears here. On the other hand, if the volume is finite, the two vacuum states are generally unitary equivalent[2].

According to Reeh-Schlieder's theorem, the vacuum is defined by the action dense in the Hilbert space containing the values $(\overline{A(D)\Omega} = \overline{A\Omega})$. In other words, it becomes a cyclic vector [13]. Moreover, it is a separable vector for the double commutative ring of the representation of the action. i.e.

Theorem 2.1 Reeh-Schlieder

For the vacuum state ψ, if we assume weakly additivity [a]. the vacuum vector Ω_ψ is the cyclic vector of $\pi(\mathcal{A}(\mathcal{D}))$ in any bounded region D, and $\pi(\mathcal{A}(\mathcal{D}))''$. is the separation vector.

[a] see [9: Def:4.13] for this.

In general, with respect to an arbitrary coordinate system and an arbitrary operator $Q \in \mathcal{A}$ that reduces the energy in that coordinate system, the state of $Q \in \ker\psi$ is called the vacuum state. And about that vacuum,

[2] An analogy to this situation is the cluster decomposition of the factor states in algebraic quantum field theory ([15] and [17]) is a situation in which the Hilbert space is discretely separated by the vacuum state, and for two spatially separated physical quantity regions D_1,D_2, the vacuum becomes in a product state, that is, a cluster state Therefore, it is not directly related to the orthogonality of the vacuum above. The spatially distant vacuum representation is noted as like $\lim_{\lambda\to\infty} \psi(Q_1(\alpha_{\lambda,\mathbf{1}}\psi(Q_2)) = \psi(Q_1)\psi(Q_2)$ for a time automorphism $\alpha_{(\lambda x,\mathbf{1})}$ of the Poincar group. This notion is asymptotic equation from the entangled state to the product state.

Theorem 2.2

(Restricted inhomogeneous Lorentz group: i.e. Poincar group $\mathcal{P}_+^{\uparrow}$ does not have to be assumed) The following are equivalent for the vacuum state ψ,
(a) The representaation $\pi_\psi(\mathcal{A})''$ is a factor; $(\pi_\psi(\mathcal{A})''\ \pi_\psi(\mathcal{A})' = \mathbf{C1})$
(b) $\pi_\psi(\mathcal{A})$ is irreducible: $(\pi_\psi(\mathcal{A})' = \mathbf{C1})$
(c) The translation invariant vector is proportional to the vacuum vector Ω_ψ.

Therefore, from the standpoint of sector theory, we can see that the (degenerate) vacuum representation (distributed) exists for each irreducible decomposition, and that $\mathcal{A}$ is separable (discrete), the vacuum state ψ can be written as a direct integral (direct sum) such that $\pi_\psi(\mathcal{A})''$ is a factor. It also follows that the vacuum representation in a given region (not spatially separated in the sense of theory of relativity) is unique. It is also possible to consider that the superselection rule is at work here. In other words, considering the previous theory of physics, it can be regarded as being separated into non-equivalent vacuum representations. However, it would depend on the type of von Neumann algebra of the irreducible representation[3].The situation is different for type I and non-type I cases.

If the irreducible representation of the vacuum representation is von Neumann type I, the cluster decomposition of the vacuum state is unique from the definition of the irreducible representation[4]. However, for finite degrees of freedom, the irreducible representation is unique than the unitary equivalence by Stone-von Neumann theorem. In other words, there is only one vacuum sector.

A more detailed analysis of the vacuum sector can be found in [17]. It assumes three conditions: relativistic causality, relativistic covariance, and energy-momentum spectral condition (Araki-Hagg locality condition), and in the intersection with each coherence subspace $(H_{0.\gamma} := H_0 \cap H_\gamma \neq 0)$, the von Neumann algebras with vacuum sector of typeI_∞ is shown to decompose into a facto[5]$(\mathcal{A} = \oplus \mathcal{A}_\gamma,\ H = \oplus H_\gamma)$. The sector of vacuum superselection law is special in the above sense.

[general sector (excitation state)]

Next, we consider an excited state from vacuum. However, not all physical quantities of excited states can be handled by the sector theory. Initially, the conditions were rather tight, but the work of Buchholz-Fredenhagen extended the charge conditions involved in the superselection rule. In general, physical quantities such as energy and momentum are global observables. In addition to the general definition of vacuum, there are important

[3]According to Haag's notation [15: p.144], If $\mathcal{U}$ is the total domain and $\mathcal{O}$ is the finite domain, then (as like $\mathcal{R}(\ \mathcal{U}) = (\cup_{\mathcal{O}} \subset \mathcal{R})''$) they are noted as double commutative algebras of the sum of the vacuum representations of the regions .

[4]When it comes to the type III argument, the vacuum representation becomes non-equivalent, but this means that the sectors are on some projections (e.g., the relation between spatial regions), rather than being compared, it probably depends on the volume of the space. This point is not very clear. This may be because the vacuum itself does not consider a space-time that is not translation invariant because the translation invariant vector is a constant multiple of the vacuum vector.

[5]Here, the I_∞ type appears when a subsystem (local region) is considered to be contained in the enclosing region. That is, when the von Neumann algebra $\mathcal{A}$ contains a weakly topological dense AF(Approximately finite) C^* algebra as a subalgebra. In this case $\mathcal{A}$ is said to be a hyperfinite von Neumann algebra or an injective von Neumann algebra [7].

criteria for how to organize deviations from the vacuum state into superselection rules. That stands for **DHR Analysis**[6]. There are basically two criteria.

1) It is a criterion for selecting meaningful localized excited states from the vacuum. If the excited states are localized in the spacetime region D, then in the relativistic causal complement D' which is not affected by D, the excitation representation π_ω gives a criterion that is an equivalent representation (unitary equivalence) to the vacuum representation π_0. i.e,

$$\pi_\omega|\mathcal{A}(D') \cong \pi_0|\mathcal{A}(D'),\ (D' : \textit{unbounded})$$

2) the requirement that the localized excitation be mobile; This means that what holds in 1) also holds for space-time translation $D + a$.

Therefore, if we add 2) to 1), let U_a be the unitary operator from the Hilbert space H_0 to H_ω, and If we set one space-time region D, for each D and its space-time translation $D_a := D + a$, any region which is causally independent of D_a, there exists a unitary operator for $D'_a \in K$, resulting in the following equality.

$$\pi_\omega(A) = U_a \pi_0(A) U_a^*,\ (A \in \mathcal{A}(D'_a))$$

From the above, it is a requirement that physical states with localizable charges excited from the vacuum (superselective charges) appear to be vacuum-like in spatial regions where the charge does not reach[7].The sector theory considers each relation and structure of the unitary equivalence class $[\pi]$ of the representations π that satisfies 1) and 2) above. The representation obtained by defining the automorphism of the representation on the domain D in this equivalence class is called the sector. In terms of physics, it is “the algebraic factor representation (irreducible representation) of the observable”.

In Araki’s text, two additional assumptions (A) and (B) are made to analyze the DHR analysis based on 1) and 2) above.

(A) Haag duality:

For any double cone region D, the following relation is required with respect to the vacuum representation π_0.

$\pi_0(A(D))' = [\bigcup\{\pi_0(\mathcal{A}(D_1)); D_1 \subset D'\}]'' \cong \pi_0(\mathcal{A}(D'))''$

(Note $D_1 \subset D'$, where D' is the causal subset of D). The last $\cong$ is according to 1) above. The symbol of (bi)commutant (bi)commutant means von Neumann ring theorem (density theorem), but what is important is the vacuum region. The duality is that considering the co-expression of the vacuum representation on D is the same as considering the vacuum representation on D'.

For the open future cone D, $D_1(\bar{D} \subset D_1)$ containing it, and any projection E of $\mathcal{A}(D)$ ($\neq 0$), there exists an isometric operator W of $\mathcal{A}(D_1)$ that satisfies $WW^* = E$. Using the vacuum representation, this property can be rewritten as follows [10]. Let π_0

[6] Doplicher-Haag-Roberts.

[7] However, in general, the electric charge responsible for the electromagnetic force is a long-range force (against relativistic causality), so it does not apply according to Gauss’s law. Generally refers to localized charge such as strong force (QCD).

be a vacuum representation, assume the above $\bar{D} \subset D_1$, and let D_2 be $D_2 \subset D' \cap D_1$, for a non-zero projection in $E \in \pi_0(D)$, we have $E \sim I$ (mod $\pi_0(D_1)$).

This Borcher property reflects the property of type III von Neumann algebras[8].

Looking at [9] and [11], the above property is the Poincar invariance and the energy condition (the analytic vector exists for the energy operator and the power action is a positive value). This point seems important when considering superselection, superposition, and split property.

1.2 Supplement: Special features of vacuum

Here, we summarize the peculiarities of the vacuum state. The state of the vacuum, if it is a degenerate state, is an important state in elementary particle theory, such as the appearance of a vacuum in the direction of breaking the symmetry when the degeneracy is loosen in spontaneous symmetry breaking.

The question of how many vacuums there are (non-equivalent vacancies), or in what state they are unique, is not currently known without taking into account the size of the Hilbert space and the nature of the operator taking the vacuum expectation. However, from the point of view of algebraic quantum theory, there is a famous theorem for the vacuum state.

In [Theorem 2.1], we arranged the Reeh-Schlider theorem as an representation based on Araki's text. In particular, the vacuum state is defined by the restricted inhomogeneous Lorentzian group: when we assume the Poincar group $\mathcal{P}_+^\uparrow$ transformations and the spectral condition (also by translation the closed set of the (forward) light cone), we have a cyclic and split vector. This is called the "standard state" of vacuum [10]. This standard representation is derived as a special case of the representation of von Neumann algebras when we consider the "Weight" is semi-finite, faithful and normal in the theory of operator algebras, which is derived as a special case [1].

Also, as a philosophical problem, if the observable algebra is spatial, the vacuum is a separating vector for each local algebra, which induces a nonlocal problem [16]. Since the discussion around here is deep and delicate, I will omit it here.

2 The Meaning of Sector Theory

2.1 Relationship with superselective charge

A sector, as mentioned at the beginning of this paper, refers to a superpositionable subspace of equivalence classes sorted by superselective charges. Traditionally, pure states are classified according to their equivalence classes. Today this sector is being analyzed in detail, including excited states, and it makes no sense to expand on the whole here. Here, I would like to organize only the points according to [9][10].

Sector theory is based on the vacuum representation. Fredenhagen calls a sector that satisfies Haag's duality a "simple sector"[13].I will explain this point briefly. On the

[8]If a von Neumann algebra M is nonfinite and has a nonzero projection, then M is of type III.

vacuum representation $\pi_0(\mathcal{A})$, we define the homomorphism ρ which defines the representation of the excited state satisfying the previous first criterion for the DHR analysis, for $A \in \mathcal{A}(D')$, as $\rho(A) = V\pi(A)V^{*}$[9]. In that case, instead of the equivalence class $[\pi]$ for the representation of excited states, we have $\rho(A) := \pi_\rho(A)$.

By this, for an excited state, an endomorphism is defined, and ρ is said to be localized when $\rho(A) = A$ (making A invariant). The support of this ρ is the area D in question. By investigation of the equivalence classes of multiple localized automorphisms and the mobility of ρ with support within the domain, the classification of excited states and the statistical properties of bosons and fermions can be explored in the sector theory (derived as a statistic method). This ρ is taken as a charged quantum number by its equivalence class, which corresponds to the superselective charge.

Returning to the vacuum representation, in this index ρ is said to be a simple sector if it is an irreducible decomposition, i.e., one-dimensional, and thus the vacuum sector is simple according to [Theorem 2.2].

2.2 Discrete Superselective Charges and Continuous Charges

We found that superselective charges are important in classifying superposition subspaces into equivalence classes by superselectition rules. However, the classification by this superselective charge is done in a discrete sense, and the classification of the representation space is also discrete sum. I.Ojima has clearly stated this point [2], and in fact, in [12], etc., the charge acting on the superselection rule must be localized, and continuous sectors are hardly considered.

2.3 From Shigeru Machida's interpretation

The requirement for continuous superselective charges associated with this continuous sector in observational problems is found in [6]. According to Machida, contraction of the wave packet occurs if we assume a continuous superselective charge between the observation devices (eg, the array of films that detect the photons). In practice, this is impossible due to the finite number of particles that make up the observation equipment, but by increasing the accuracy, it is possible to observe the approximate contraction of the wave packet. In other words, if the distribution of the particles in the film that detects the photons is concentrated in one place like a delta function, the contraction of the wave packets will not occur and a quantum interference effect called superposition will be found. become.

This is related to what is written in [8], so I will explain it briefly. From the point of view of algebraic quantum field theory, the superposition of wave functions (state functions) is the superposition of "two states ω_1 and ω_2 and "The GNS representations π_{ω_1} and π_{ω_2} associated with each are not disjoint intertwitting maps containing unitary maps"[10] or "they are not unrelated representations" are equivalent .That is, if two rep-

[9] V is a unitary operator and the right-hand side is bounded: $V\pi(A)V^* = A$: a unitary operator from the representation space to the Hilbert space.

[10] In the case of irreducible representations, unitary non-equivalence is meaningful, but we also consider cases where it is not. See [2] for this.

resentations are not disjoint, they will not be separated into different subspaces, since no orthogonal projections are attached to the representations. If a representation $\pi(A)$ is an irreducible representation, then u satisfying $[u.\pi(A)] = 0$ is $CI = \pi(A)'$, which represents superposition (coherent).

Machida's explanation seems to be somewhat confusing because there is no type classification of von Neumann algebras. Here, the superposition, limited to finite particle systems, is described by a von Neumann algebras of type I, which (from the Stone-von Neumann uniqueness theorem) is the sector is determined to be one[11].

If we accept the continuous superselective charge, does it differ from the explanation when the superselective charge exists discretely? If it exists discretely, in a finite particle system, the uniqueness theorem says that superposition always requires one sector, so there will be no contraction of the wavepacket. However, the observer is a classical system, and in general the product state of the quantum system and the classical system should give rise to a mixed state. A spot of light on the film and a trace of a spot on another film gradually build up shadows and show interference fringes, which could be interpreted as all the same sectors being observed in similar conditions. It is possible. Since each of $\pi_i(A)$,$(i \in N)$ representing infinite light points is unitary equivalent, it is interpreted that it is preserved even after observation. There is also the possibility that it could be done.

It seems that Machida brings up the continuous charge here in order to break this unitary equivalence. But in fact, it would rather have to first set the type of von Neumann algebra, or the infinite quantum system. In the case of type III, which commonly appears in infinite particle systems, even if a continuous superselective charge can be introduced, only a mixed state can be expected from the point of view of the traditional superselection rule. If it is a finite particle system, it can be understood as type I, but in this case as well, if we do not assume the mixed phase described by Ojima [2: 45], which is an intermediate position between the pure state and the mixed state, interference will occur in an asymptotic sense. The "observation" of the effect becomes incomprehensible. Machida's explanation was somewhat unclear on this point. However, the argument would change if matrix elements (as order parameter) were taken into account, which indicate changes in the number of particles as like in the superfluid phase transition.

To request continuous superselective charges or continuous sectors, a different approach seems necessary. In [2:37page], there is a clear interpretation; "we does not avoid the appearance of continuous sectors in algebraic quantum field theory and we incorporates "order parameter" into the sector theory".

2.4 Izumi Ojima's Inquiry

Ojima's proposal itself is written in [3], but it may seem a bit difficult to those who come into contact with this theory for the first time. So if you take out only the points by replacing the bones, it will be as follows. Extending the view of the pure state from the standpoint of quantum field theory to the "pure phase" represented by the thermodynamically pure phase, and the mixed state to the complementary concept of "mixed phase".

[11] As seen in the Aharonov-Bohm effect, etc., there are exceptions that violate the uniqueness theorem depending on external conditions such as boundary conditions.

It's a way of looking at it.

This view may not be just to connect quantum field theory to the concepts of quantum statistical mechanics. Rather, it is a reversal of our way of thinking, bound by the framework of quantum physics of finite particle systems. Finite particle systems basically deal only with the world of type I von Neumann algebras. Then, the story ends with the irreducible decomposition (representation) and reducible decomposition (representation) of the expression $\pi(A)$. However, in the case of type III, which usually appears in infinite particle systems, the representations are in principle non-equivalent to each other when looking at the irreducible representations from the beginning. It may be in thermal equilibrium where it cannot be decomposed further. Considering that the irreducible expression as mathematics is not directly linked to the thermal equilibrium state in physics, the concept is expanded to cover the thermal state (mixed state).

Along with this, a macro index, i.e. the order parameter, is derived for superselective charges and discrete ordinary sectors. An order parameter is generally an order parameter possessed by the "phase" of a physical system [14]. For example, "density" is one of the indices for distinguishing the phases of substances (distinguishing phase transitions) such as liquid, gas, and solid. In a magnetic material, macroscopically, it is a magnetized structure, but since it is possible today by adding magnetic moments, the microscopic spin becomes important. As an order parameter that appears in spontaneous symmetry breaking in elementary particle theory [4], it is related to the conserved quantity Q when calculating the vacuum expectation value. If the symmetry is not broken, $Q|0>= 0$ for the vacuum state $|0>$, and formally, $Q|0>\neq 0$ is broken. But on the divergence of the expected value, we consider the commutation relation $i[Q, A(x)] = \delta_Q A(x)$[12], with the local operator $A(x)$ as an index, and this $\delta_Q A(x)$ is now the order parameter ($\delta_Q A(x)|0>\neq 0$ be the index of spontaneous symmetry breaking.).

While the order parameter has become a macro index in thermodynamics, today it also has a connection with the micro. Also, due to the nature of the index, it is basically a continuous parameter. By using this as a sector classification index, continuous sectors can be understood. In addition, as Machida argued, the difficulties of conventional observational theory can also be understood consistently. Through this discussion, Ojima suggests that pure phases (single-sector internal structure understanding: peculiar states of quantum systems: pure coherence), mixed phases (probabilistic mixing of multiple sectors: quantum systems and observational systems (macroenvironmental systems) coexistence with, superselection rule (conventional mixed macrostate, but with order parameter).

3 Concluding Remarks: From the Viewpoint of Factor

Finally, we will organize what has been said so far from the point of view of factor.

For von Neumann ring $\mathcal{A}$, let $\mathcal{A}\cap\mathcal{A}'$ be the center of $\mathcal{A}$ ($Z(\mathcal{A})$) So, if $Z(\mathcal{A}) = \mathbf{C}1$, then $\mathcal{A}$ is said to be a factor. Factor provide an important way of classifying von Neumann rings through factor decomposition of von Neumann algebras [1], so to consider factor is

[12]This commutation relation follows from the infinitesimal transformation of Noether's theorem.

to consider properties of the von Neumann algebras in question. be the same.

By the way, when I organized the uniqueness of the vacuum earlier, I mentioned vacuum degeneracy and types of algebras. Using the theory of irreducible decompositions of algebra, from the standpoint of algebraic quantum field theory, "a vacuum representation has a unique vacuum state Ω" is "an irreducibility of a vacuum representation". Furthermore, "the vacuum representation is the same as $\mathcal{A} = \mathcal{B}(\mathcal{H})$: the right-hand side is a bounded linear operator on some Hilbert space". This is the case when $\mathcal{A}$ is a factor.

The irreducible decomposition is based on the quantum physical observational algebra with finite degrees of freedom in the case of von Neumann algebras of type I. In this case, the irreducible decomposition becomes unique, as we have said many times. Therefore, in the case of type I, vacuum representations are all unitary equivalents, since in principle there is only one irreducible representation. However, when it comes to infinite degrees of freedom, there are an infinite number of different irreducible representations. Therefore, since the equivalence between one irreducible representation and another irreducible representation does not hold, we have representations of degenerate vacuums, and so on. In other words, in the case of non-I type, even if we take irreducible representations, we cannot connect each representation. No further analysis possible (The so-called "abundance of irrelevance": the covariance of vacuum expressions such as Goldstone's theorem is broken)[13]. Therefore, in the case of non-type I, it is important to analyze states with non-trivial centers ($Z(\mathcal{A}) \neq \mathbf{C}1$) as Ojima mentioned[14]. And according to Gelfand's representation theorem, a commutative algebra is equivalent to a continuous function ring on a Hausdorff space. Therefore, from the viewpoint of spectral analysis, it is important to diagonalize the non-trivial center and look at its structure algebraically.

As I wrote in the opening footnote, it is important to think of sector theory as "decomposition into factor representations" rather than based on irreducible decomposition, and I-type irreducible decomposition is an exception. , is important to understand. In other words, when we consider the decomposition of the center ($Z(\mathcal{A})$), it is better to understand the sector as a "partial representation" of each decomposed (in a broad sense) in the non-I-type world (because most real world are non-type I).

[13] Note; the Reeh-Sclider theorem is not violated. Since irreducibility means the smallest unit of decomposition, the vacuum is unique, cyclic and separable in some smallest vacuum representation. There are (infinitely many) irreducible representations in non-I types. Also, the vacuum representation and the vacuum vector are different concepts. However, since the vacuum representation shows how to take the expected value, it is possible to interpret that the vacuum vector is different. After all, if we do not observe something about the vacuum, we cannot obtain the vacuum expected value (observed value). Since the vacuum vector is interpreted as a state in which there are no particles in Fock space, it seems physically strange that it differs from place to place. Also, the vacuum cannot be considered to be the same state spreading. There are worlds other than vacuum (excited worlds) everywhere in this world. So what exactly is a pure vacuum remains a mystery. It should be emphasized, however, that in algebraic quantum field theory the Hilbert space need not be set up first. A vector is constructed from the observational algebra (GNS construction method). At this time, a vacuum vector is required as a cyclic vector (as a minimum requirement). Physics is meaningless without taking into account the means of observing the vacuum.

[14] Ojima describes factor representation in [2]. The importance of mixed phase analysis.

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