Analysis of Asymmetrical Three-Phase Circuits

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ABSTRACT

This paper develops spiral vector symmetrical component method for analyzing AC asymmetrical three-phase circuits. It has been shown that we can get general analytical solution including steady and transient state for the lumped constant circuit. A numerical example is also given in the paper. The conclusion is that the new method can successfully be applied to transient analysis of AC circuits.

1. INTRODUCTION

Conventional electric circuit theory is in difficulties in AC transient analysis. It is troublesome to combine results of AC steady analysis with transient’s because there are two different variable expressions for the same circuit, that is, phasor notation and instantaneous value expression. Nevertheless, instantaneous value symmetrical component method is lacking in mathematical basis because its positive-sequence component is conjugate to its negative’s [1]. Fortunately, as a new AC circuits and machines theory, spiral vector theory was proposed by Dr. Yamamura and it is receiving great success [1]. It is expected that spiral vector theory can overcome difficulties in AC transient analysis.

With spiral vector theory, all variables are expressed as spiral vectors that are rotating counterclockwise in the complex plane. A spiral vector is shown in Fig. 1 and its expression is

\[ x = Ae^{\delta t}, \quad \delta = -\lambda + j\omega \]  

(1-1)

From Eq. 1-1, we see that spiral vectors can not only express steady AC (\( \lambda = 0 \)), DC (\( \lambda = 0, \omega = 0 \)) variables, but also transient AC and DC (\( \omega = 0 \)) variables. Therefore spiral vectors can express almost all variables in electric circuits. Figure 2 illustrates scopes of electric circuit’s variable. In Fig. 2, set \( P \) is the sum of phasor variables, set \( R \) is the sum of instantaneous values; in the former AC steady analysis, in the latter AC transient analysis is carried out, respectively. Set \( C \) is the sum of spiral vector variables, in which spiral vector method is carried out. Staring at Fig. 2, we see that there are larger area is still left untouched by conventional theory because set \( C \) is bigger than set \( P \) plus set \( R \). The left area is AC transient analysis with spiral vector variables.

![Figure 1 A spiral vector](image)

![Figure 2 Scopes of electric circuit’s variable](image)
cludes conclusions for this paper.

2. SPIRAL VECTOR SYMMETRICAL COMPONENT METHOD

Consider a star connection asymmetrical three-phase network of Fig. 3. We will solve this network with spiral vector symmetrical component method.

![Figure 3](image-url) An asymmetrical three-phase network

Flow chart for spiral vector symmetrical component method solving the problem is shown in Fig. 4. We will explain it in detail.

![Figure 4](image-url) Flow chart for spiral vector symmetrical component method

**Step 1:** Write circuit’s differential equations.

Assuming $v_a$, $v_b$, $v_c$ are phase voltages, $i_a$, $i_b$, $i_c$ are phase currents and $i_N$ is current of neutral line, where subscript $a$, $b$, $c$, $N$ stand for $a$, $b$, $c$ phases and neutral line respectively. Thus

\[ i_N = i_a + i_b + i_c \]  

(2-1)

Here all variables are spiral vectors.

Step 2: Make symmetrical transformations.

We assign

\[ \alpha = e^{-j2\pi/3}, \quad \alpha^2 = e^{j2\pi/3}, \quad \alpha^3 = 1 \]  

(2-5)

and

\[ A^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \]  

(2-6)

$A$ is called as the transformation matrix and $A^*$ is the conjugate to $A$, in converse transformation the following relations hold:

\[ A^{-1} = \frac{1}{3} A^* \]  

(2-7)

We assume $i_{0}, i_1, i_2$ and $v_0, v_1, v_2$ are zero-, positive-, negative-sequence component currents and voltages, and write following symbols for further proceeding

\[ [i_{sym}] = \begin{bmatrix} i_0 \\ i_1 \\ i_2 \end{bmatrix} \]  

\[ [v_{sym}] = \begin{bmatrix} v_0 \\ v_1 \\ v_2 \end{bmatrix} \]  

(2-8)

where subscript $sym.$ stands for symmetrical component. Then transforming three-phase currents and voltages into symmetrical components described as

\[ [i] = A^*[i_{sym}] \]  

\[ [v] = A^*[v_{sym}] \]  

(2-9)

Substituting Eqs. 2-1 and 2-9 into 2-3,

\[ [C(p)][i_{sym}] = [D(p)][v_{sym}] \]  

(2-10)

can be obtained. Where

\[ [C(p)] = A([A(p)]A^*) + A([A_N(p)]A^*) \]  

\[ [D(p)] = A([B(p)]A^*) \]  

(2-11)
Step 3: Segregate symmetrical component
Segregating the zero-, positive- and negative-sequence component equations from Eq. 2-10, and assuming different symmetrical components are independent each other, we obtain

\[ Z_N(p) i_0 = T_0(p) v_0 \]
\[ Z_0(p) i_1 = T_0(p) v_1 \]
\[ Z_0(p) i_2 = T_0(p) v_2 \]

(2-12)

where \( Z_0(p), Z_N(p) \) are polynomials in \( p \) shown in

\[ Z_0(p) = z_{00} + z_{01} p + \ldots + z_{0n} p^n \]
\[ Z_N(p) = z_{N0} + z_{N1} p + \ldots + z_{Nn} p^n \]

(2-13)

We call \( Z_0(p) \) as symmetrical component polynomial, \( Z_N(p) \) as zero-sequence component polynomial. If there are no elements in neutral line, \( Z_0(p) \) is equal to \( Z_0(p) \).

Step 4: Obtain steady state solutions
Assuming input voltages are circular vectors (steady AC), \( p \) becomes \( j \omega \). The steady state solutions are obtained as

\[
\begin{bmatrix}
  i_0 \\
  i_1 \\
  i_2
\end{bmatrix} = \left[ D(j \omega) \right] \begin{bmatrix}
  V_0 \\
  V_1 \\
  V_2
\end{bmatrix}
\]

(2-14)

Here we see that the solutions are also circular vectors.

If input voltages are spiral vectors, the spiral vector steady state solutions are yielded as

\[
\begin{bmatrix}
  i_{0s} \\
  i_{1s} \\
  i_{2s}
\end{bmatrix} = \left[ D(\delta) \right] \begin{bmatrix}
  V_0 \\
  V_1 \\
  V_2
\end{bmatrix}
\]

(2-15)

where subscript \( s \), \( l \) and \( 2s \) stand for spiral vector steady state solutions for zero-, positive- and negative-sequence component.

Step 5: Solve characteristic equations
From Eqs. 2-12 and 2-13, we get symmetrical component and zero-sequence component characteristic equations as

\[ Z_0(p) z_00 + z_{01} p + \ldots + z_{0n} p^n = 0 \]
\[ Z_N(p) z_{N0} + z_{N1} p + \ldots + z_{Nn} p^n = 0 \]

(2-16)

The positive- and negative-sequence component characteristic equation is the same. This the evidence that symmetrical component method satisfied principle of superposition.

Let characteristic roots of Eq. 2-16 are

\[
\begin{bmatrix}
  \delta_{01} \\
  \delta_{02} \\
  \ldots \\
  \delta_{0n} \\
  \delta_{11} \\
  \delta_{12} \\
  \ldots \\
  \delta_{1n}
\end{bmatrix}
\]

(2-17)

then we get transient solutions

\[
\begin{align*}
  i_0 &= A_{00} e^{\delta_{01} t} + A_{02} e^{\delta_{02} t} + \ldots + A_{0n} e^{\delta_{0n} t} \\
  i_1 &= A_{11} e^{\delta_{11} t} + A_{12} e^{\delta_{12} t} + \ldots + A_{1n} e^{\delta_{1n} t} \\
  i_2 &= A_{21} e^{\delta_{21} t} + A_{22} e^{\delta_{22} t} + \ldots + A_{2n} e^{\delta_{2n} t}
\end{align*}
\]

(2-18)

where \( A_{00}, A_{02}, \ldots, A_{0n}, A_{11}, A_{12}, \ldots, A_{1n}, A_{21}, A_{22}, \ldots, A_{2n} \) are arbitrary constants, which are to be determined by initial conditions. If there are \( n \) multiple roots among characteristic roots, the following terms should be included in Eq. 2-18.

\[
\begin{align*}
  A_{01} t e^{\delta_{01} t} + A_{02} t^2 e^{\delta_{02} t} + \ldots + A_{0(n-1)} t^{n-1} e^{\delta_{0n} t} \\
  A_{11} t e^{\delta_{11} t} + A_{12} t^2 e^{\delta_{12} t} + \ldots + A_{1(n-1)} t^{n-1} e^{\delta_{1n} t} \\
  A_{21} t e^{\delta_{21} t} + A_{22} t^2 e^{\delta_{22} t} + \ldots + A_{2(n-1)} t^{n-1} e^{\delta_{2n} t}
\end{align*}
\]

(2-19)

Step 6: Evaluate arbitrary constants
If cases that input voltages are circular vectors are treated, general symmetrical component solutions are

\[
\begin{bmatrix}
  i_{0r} \\
  i_{1r} \\
  i_{2r}
\end{bmatrix} = \begin{bmatrix}
  \sqrt{2} I_0 \\
  \sqrt{2} I_1 \\
  \sqrt{2} I_2
\end{bmatrix}
\]

(2-20)

Substituting initial conditions into Eq. 2-20, we can evaluate arbitrary constants of the equation.

Substituting Eq. 2-20 into 2-9, the general three-phase solutions can be obtained.

Step 7: Calculate instantaneous values
Finally, the instantaneous value component solutions are given by

\[
\begin{bmatrix}
  i_{a, real} \\
  i_{b, real} \\
  i_{c, real}
\end{bmatrix} = \begin{bmatrix}
  \text{Re}(i_a) \\
  \text{Re}(i_b) \\
  \text{Re}(i_c)
\end{bmatrix}
\]

(2-21)

where \( \text{Re} \) means real part of the complex number, subscript \( \text{real} \) means real part of spiral vectors.

The author will investigate AC power in one other paper, here only the solutions are given. The instantaneous active power \( p \) and reactive power \( q \) in single phase are defined as

\[ p = (1/2) \text{Re}(vi^* + vi) \]
\[ q = (1/2) \text{Im}(vi^* + vi) \]

(2-22)

where \( i^* \) means the conjugate to \( i \). We call \( vi^* \) as difference-frequency power and \( vi \) as quasi-frequency power. In steady state, the difference-frequency power is equal to RMS(Root-Mean-Square) power.

We can also get three-phase difference-frequency active power \( P_f \) and reactive power \( Q_f \), as

\[
\begin{bmatrix}
  P_f \ &= (3/2) \text{Re}(v_i l_0^* + v_i l_1^* + v_i l_2^*) \\
  Q_f \ &= (3/2) \text{Im}(v_i l_0^* + v_i l_1^* + v_i l_2^*)
\end{bmatrix}
\]

(2-23)

We call above procedure as spiral vector symmetrical component method. In AC steady analysis, it can inherits all the work of phasor analysis theory; in AC
transient analysis, analytical solutions can be obtained. Moreover, we see that with the solutions, positive- and negative-sequence component are independent, so the new method is better than instantaneous value symmetrical component method.

There is a question should be answered here. “What is relationship between spiral vector method to Lapalace transformation?” The answer is that if we use spiral vector variable, the solution is the same. But we have not need Lapalace transformation here because its procedure is more complicated than spiral vector method.

If elements of the circuits are non-lumped constant, we have to take numerical calculation. However, spiral vector variable should also be used in simulation. Although complex number calculation is time-consuming than real number, we get rewards, for example, transient power of the circuit shown in Eq. 2-22. The author is studying how to simulate non-lumped constant circuits. In the next section, we’ll study a practical asymmetrical three-phase LCR circuit.

3. SOLVING AN ASYMMETRICAL THREE-PHASE CIRCUIT

3.1 Differential Equations of the Circuit

Consider a star connection asymmetrical three-phase LCR circuit of Fig. 5. At t=0, switch S is closed, calculate three-phase currents and three-phase difference-frequency active and reactive power. We use previously proposed method to solve this problem.

Figure 5 An asymmetrical three-phase LCR circuit

Assuming instantaneous values of input voltages as

\[
\begin{align*}
e_a &= \sqrt{2}E_a e^{j(\omega t + \varphi_a)} \\
e_b &= \sqrt{2}E_b e^{j(\omega t + \varphi_b)} \\
e_c &= \sqrt{2}E_c e^{j(\omega t + \varphi_c)}
\end{align*}
\]  

(3-1)

where \(E_a, E_b, E_c\) are effective values of phase input voltages, \(\varphi_a, \varphi_b, \varphi_c\) are phase angles.

Spiral vector theory always uses cosine function for steady state instantaneous value because it can bring much convenient for analysis, any sines are to be converted to cosines before proceeding further.

The spiral vector expressions of input voltages can be written as

\[
\begin{align*}
e_a &= \sqrt{2}E_a e^{j(\omega t + \varphi_a)} \\
e_b &= \sqrt{2}E_b e^{j(\omega t + \varphi_b)} \\
e_c &= \sqrt{2}E_c e^{j(\omega t + \varphi_c)}
\end{align*}
\]  

(3-2)

The general differential equation for the time period after the switch S is closed can be obtained by writing Kirchhoff’s voltage law around the circuit.

\[
\begin{align*}
&j \omega e_a = L_a e_a^2 + [R_a p q_a] + \left[ q_a / C_a \right] \\
&j \omega e_b = L_b e_b^2 + [R_b p q_b] + \left[ q_b / C_b \right] \\
&j \omega e_c = L_c e_c^2 + [R_c p q_c] + \left[ q_c / C_c \right]
\end{align*}
\]  

(3-3)

where \(q_a, q_b, q_c\) are charges of a, b, c phases and neutral line, \(p\) means \(d/dt\), \(q^2\) means \(d^2/dt^2\). Thus \(q_N = q_a + q_b + q_c\).

In Eq. 3-5, we do symmetrical transformations for input voltages \(e_a, e_b, e_c\) and charges \(q_a, q_b, q_c\). If inserting Eqs. 3-4 and 3-5 into Eq. 3-3, equation 3-6 can be obtained.

\[
\begin{align*}
e_a &= \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\
\end{bmatrix} \begin{bmatrix} e_0 \\ e_1 \\ e_2 \end{bmatrix} \\
e_b &= \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\
\end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \\ q_2 \end{bmatrix} \\
e_c &= \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\
\end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \\ q_2 \end{bmatrix}
\end{align*}
\]  

(3-5)

\[
\begin{align*}
th[e_{sym}] & = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\
\end{bmatrix} \begin{bmatrix} e_0 \\ e_1 \\ e_2 \end{bmatrix} \\
th[q_{sym}] & = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\
\end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \\ q_2 \end{bmatrix}
\end{align*}
\]  

(3-6)

where \([e_{sym}], [q_{sym}], [L_{sym}], [R_{sym}], [1/C_{sym}]\) are given as Eqs. 3-7 through 3-11.

\[
\begin{align*}
e_{sym} &= \begin{bmatrix} e_0 \\ e_1 \\ e_2 \end{bmatrix} \\
q_{sym} &= \begin{bmatrix} q_0 \\ q_1 \\ q_2 \end{bmatrix}
\end{align*}
\]  

(3-7)

\[
\begin{align*}
L_{sym} &= \begin{bmatrix} l_0 + 3L_N \\ l_0 \\ l_0 \\
\end{bmatrix}
&= \begin{bmatrix} r_0 + 3R_N \\ r_0 \\ r_0 \\
\end{bmatrix}
&= \begin{bmatrix} 1/c_0 + 1/C_N \\ 1/c_0 \\ 1/c_0 \\
\end{bmatrix}
\end{align*}
\]  

(3-8)

\[
\begin{align*}
R_{sym} &= \begin{bmatrix} l_0 + 3L_N \\ l_0 \\ l_0 \\
\end{bmatrix}
&= \begin{bmatrix} r_0 + 3R_N \\ r_0 \\ r_0 \\
\end{bmatrix}
&= \begin{bmatrix} 1/c_0 + 1/C_N \\ 1/c_0 \\ 1/c_0 \\
\end{bmatrix}
\end{align*}
\]  

(3-9)

10)

The symmetrical component inductors, resistors and capacitors are

\[
\begin{align*}
l_0 &= (1/3)(L_a + L_b + L_c) \\
l_1 &= (1/3)(L_a + \alpha L_b + \alpha^2 L_c) \\
l_2 &= (1/3)(L_a + \alpha^2 L_b + \alpha L_c)
\end{align*}
\]  

(3-3)
12) \[ r_0 = (1/3)(R_o + R_p + R_c) \]
\[ r_1 = (1/3)(R_o + \alpha R_p + \alpha^2 R_c) \]
\[ r_2 = (1/3)(R_o + \alpha^2 R_p + \alpha R_c) \]  
(3-12)

13) \[ 1/c_0 = (1/3)(1/C_a + 1/C_b + 1/C_c) \]
\[ 1/c_1 = (1/3)(1/C_a + \alpha/C_b + \alpha^2/C_c) \]
\[ 1/c_2 = (1/3)(1/C_a + \alpha^2/C_b + \alpha/C_c) \]  
(3-13)

14) Now we segregate Eq. 3-6 and get zero-, positive-, negative-sequence component voltages equations as:
\[ j\omega e_0 = \left( l_0 + 3L_N \right) p^2 + (r_0 + 3R_N) p + \frac{1}{c_0} + \frac{3}{C_N} \]  
\[ + (l_1 p^2 + n_1 p + \frac{1}{c_1}) q_1 + (l_2 p^2 + r_2 p + \frac{1}{c_2}) q_2 \]  
(3-14)

15) \[ j\omega e_1 = \left( l_1 p^2 + r_1 p + \frac{1}{c_2} \right) q_0 \]
\[ + (l_0 p^2 + n_0 p + \frac{1}{c_0}) q_1 + (l_2 p^2 + r_2 p + \frac{1}{c_1}) q_2 \]  
(3-15)

16) \[ j\omega e_2 = \left( l_2 p^2 + n_2 p + \frac{1}{c_1} \right) q_0 \]
\[ + (l_0 p^2 + n_0 p + \frac{1}{c_0}) q_1 + (l_2 p^2 + r_2 p + \frac{1}{c_2}) q_2 \]  
(3-16)

3.2 The Steady State Solutions

For steady state, \( \rho \) becomes \( j\omega \). The steady state solutions can be obtained from Eq. 3-6 as:
\[ \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \left( j\omega \right) \begin{bmatrix} L_{sym} \\ R_{sym} \\ \left( 1 - j\omega \right) C_{sym} \end{bmatrix} \begin{bmatrix} \hat{E}_u \\ \hat{E}_r \\ \hat{E}_c \end{bmatrix} \]  
(3-17)

18) There are some differences between the solutions with conventional theory’s. With Eq. 3-18, the solutions are circular vectors rotating counter-clockwise in the complex plane, that is, they are functions of time and not stationary vectors as in phasor analysis theory. It is depending this idea that results of AC steady analysis can combine with AC transient analysis smoothly.

3.3 The Transient State Solutions

A comparison of Eqs. 2-12, 3-16 and 3-17 reveals that the symmetrical component characteristic equation is
\[ l_0 \delta^2 + n_0 \delta + \frac{1}{c_0} = 0 \]  
(3-19)

where \( l_0, r_0, c_0 \) are determined by Eqs. 3-12 through 3-14, and all of them are real numbers.

In this paper, we treat cases that satisfy
\[ r_0^2 - 4l_0/c_0 > 0 \]  
(3-20)
The roots are calculated by
\[ \delta_{1,12} = \frac{-r_0 \pm \sqrt{r_0^2 - 4l_0/c_0}}{2l_0} \]  
(3-21)

From Eqs. 2-12 and 3-15, we get zero-sequence component characteristic equation as:
\[ (l_0 + 3L_N) \delta_0^2 + (r_0 + 3R_N) \delta_0 + \frac{1}{c_0} + \frac{3}{C_N} = 0 \]  
(3-22)

If there are no elements in the neutral line, Eq. 3-22 is same as Eq. 3-19.
Assuming roots of Eq. 3-19 are \( \delta_{11}, \delta_{12} \), of Eq. 3-22 are \( \delta_{11}, \delta_{12} \), we get the transient solutions
\[ \begin{bmatrix} q_0 \\ q_{11} \\ q_{21} \end{bmatrix} = \begin{bmatrix} A_{01}e^{\delta_{11}t} + A_{02}e^{\delta_{12}t} \\ A_{11}e^{\delta_{11}t} + A_{12}e^{\delta_{12}t} \\ A_{21}e^{\delta_{11}t} + A_{22}e^{\delta_{12}t} \end{bmatrix} \]  
(3-23)

Here \( A_{01}, A_{02}, A_{11}, A_{12}, A_{21}, A_{22} \) are arbitrary constants which are to be determined by initial conditions.
We see that characteristic equation of asymmetrical three-phase \( LCR \) circuit is a quadratic equation that is same as single-phase’s.

3.4 The General Solutions

Combining steady state and transient state solutions, the general solutions are given as:
\[ \begin{bmatrix} q_0 \\ q_{11} \\ q_{21} \end{bmatrix} = \begin{bmatrix} \sqrt{2}I_0/j\omega + A_{01}e^{\delta_{11}t} + A_{02}e^{\delta_{12}t} \\ \sqrt{2}I_1/j\omega + A_{11}e^{\delta_{11}t} + A_{12}e^{\delta_{12}t} \\ \sqrt{2}I_2/j\omega + A_{21}e^{\delta_{11}t} + A_{22}e^{\delta_{12}t} \end{bmatrix} \]  
(3-24)

Substituting Eq. 3-25 into 2-9, the general three-phase currents are obtained.
The initial conditions are
\[ \begin{bmatrix} q_{1a} \\ q_{1b} \\ q_{1c} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \]  
(3-26)

In symmetrical transformation the following relations hold:
Assuming input voltages are circular vectors is taken. Next section we will give a numerical example of the circuit shown in Fig. 5.

4. A NUMERICAL EXAMPLE

Now a numerical example will be shown. The parameters of the circuit of Fig. 5 are given in Table 1. Here the case that input voltages are symmetrical circular vectors is taken.

Table 1  The parameters of the circuit

<table>
<thead>
<tr>
<th>E(V)</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(Hz)</td>
<td>50</td>
</tr>
<tr>
<td>inductor (mH)</td>
<td>L_a</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>resistor (Ω)</td>
<td>R_a</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>capacitor (mF)</td>
<td>C_a</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
</tr>
</tbody>
</table>

Assuming input voltages are

\[
\begin{bmatrix}
q_0 \\
q_1 = 1 \\
q_2 = 1 \\
i_0 = 1 \\
i_1 = 1 \\
i_2 = 1 \\
\end{bmatrix}
\begin{bmatrix}
q_a \\
q_a = 1 \\
q_a = 1 \\
i_a = 1 \\
i_a = 1 \\
i_a = 1 \\
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

Inserting Eq. 3-27 into 3-24 and 3-25, we get

\[A_01 + A_{02} = -\sqrt{2}l_0 + j\omega \]
\[\delta_{01}A_{01} + \delta_{02}A_{02} = -\sqrt{2}l_0 \]

From these equations, we can determine arbitrary constants. Next section we will give a numerical example of the circuit shown in Fig. 5.

Substituting Eq. 4-2 into 2-9 to yield three phase steady currents

\[
\begin{bmatrix}
i_a \\
i_b \\
i_c \\
\end{bmatrix} = \begin{bmatrix}7.03e^{j(\omega t + 0.0044)} \cr 55.62e^{j(\omega t - 0.0174)} \cr 18.88e^{j(\omega t + 0.4666)} \end{bmatrix}
\]

Thus we get general analytical symmetrical component solutions for the circuit

\[
\begin{bmatrix}
i_0 \\
i_1 \\
i_2 \\
\end{bmatrix} = \begin{bmatrix}9.94e^{j(\omega t + 0.0044)} + 286e^{-78.364 - j2.603} + 1119e^{-921.645 + j2.9015} \cr 78.65e^{j(\omega t - 0.1474)} + 8.58e^{-315.5 - j1.4045} + 81.71e^{-968.455 + j2.8941} \cr 26.70e^{j(\omega t + 0.4646)} + 291e^{-315.5 - j0.7925} + 27.73e^{-968.455 - j2.777} \end{bmatrix}
\]

We can obtain spiral vector currents of three-phase. We calculate three-phase difference-frequency active and reactive power of the circuit according to Eq. 2-23.

Here several graphs of results will be shown. Figure 6 shows the spiral vector currents of symmetrical component. Where symbols zero, positive, negative stand for zero-, positive- and negative-sequence component current respectively. Here we see that positive- and negative-sequence component current are independent, there are no conjugated relationship between them like instantaneous value symmetrical component method. Figure 7 shows the spiral vector currents of three-phase. All currents start from zero and stabilize in circular vectors which are rotating counter-clockwise in the complex plane.

The negative-sequence component current is rotating counter-clockwise in the complex plane same as positive’s. It is called negative-sequence component because of space order, not because of relationship between positive’s.
Figure 6  The spiral vector currents of symmetrical component

Figure 7  The spiral vector currents of three-phase

Figure 8, 9 show the instantaneous value currents of symmetrical component and of three-phase. They are the real part of the spiral vector currents.

If a field test is done, curves recorded in an oscillograph are assumed agreeing with results in Fig.9.

Figure 10, 11 show the curve of difference-frequency active and reactive power. Where symbol $d-f$ stands for difference-frequency. As $t$ grows larger, the transient diminishes and disappears, leaves only the steady state in which difference-frequency power is equal to RMS power.
5. CONCLUSIONS

This paper developed spiral vector symmetrical component method for analyzing asymmetrical three-phase circuits according to spiral vector theory. Since spiral vectors are introduced into circuit’s analysis, the general analytical solutions including steady and transient state for the lumped constant circuit can be obtained. For a star connection circuit example, we gave very detailed procedure to solve it. The numerical example and diagrams show that the proposed method is effective to AC transient analysis. For other types connection circuits, it can also get analytical solutions with familiar procedure. The author considers that AC transient analysis should switch to spiral vector theory because it is better than conventional theory.

In a nutshell, variable expression is the starting point of circuit’s analysis and the slogan of spiral vector theory is: “Same Circuit, Same Variable.” This is the key to understand this new theory.

6. ACKNOWLEDGEMENT

The author would like to acknowledge Dr. Yamamura for his guidance in spiral vector theory.

7. REFERENCES


8. BIOGRAPHY

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