Development of Integral Method for Measuring RMS Active and Reactive Power in Single- and Multiphase Networks

Kempei Seki
Mitsubishi Electric Corporation
Japan

Abstract

This paper develops integral method for measuring RMS active and reactive power in single- and multiphase networks according to spiral vector theory. At first, The paper introduces integral method to measure RMS active and reactive power in single-phase networks. Then the paper applies method not only for symmetrical three-phase networks but also for unsymmetrical three-phase networks. At last, a numerical example of unsymmetrical three-phase circuit is shown. The conclusion is that the new method is corrective and practical.

Keywords

Active power, Instantaneous power, Root-Mean-Square power, Power measurement, Power quality, Reactive power, Spiral vector theory

1 INTRODUCTION

Spiral vector theory is new AC machines and circuits theory its state variables are spiral vector rotating counter-clockwise in the complex plane [1]. With spiral vector theory, we can define instantaneous reactive power in single-phase networks first time [2]. The new definition gives possibility to develop an integral measurement of RMS reactive power in single-phase networks. On the other hand, with spiral vector theory we can also extend method of symmetrical coordinates from phasors state variables that are stationary complex numbers to spiral vector state variables that are time-varying complex numbers. The merit of the extension is that we can define and measure instantaneous and RMS active and reactive of symmetrical phase sequence in unsymmetrical three-phase networks.

Therefore, we develop integral method for measuring RMS active and reactive in single- and multiphase networks in this paper.

The paper is organized as follows. In section 2, we develop integral method for measuring RMS active and reactive power in single-phase networks. In section 3, we develop integral method for measuring RMS active and reactive power in symmetrical and unsymmetrical three-phase networks. In section 4, we show a numerical example. In section 5, we give conclusion of the paper.

2 MEASURE RMS ACTIVE AND REACTIVE POWER IN SINGLE-PHASE NETWORKS

This section introduces integral method for measuring RMS active and reactive power in single-phase networks.

2.1 Define Instantaneous Active and Reactive Power in Single-Phase Networks

The voltage in single-phase networks is expressed as

$$v = \sqrt{2}Ve^{j(\omega t+\varphi)} \tag{1}$$

where V is RMS value, φ is initial phase angle, ω is angular frequency respectively. The real and imaginary components of the voltage are expressed as

$$\left.\begin{aligned} v_{re} &= \mathrm{Re}(v) = \sqrt{2}V\cos(\omega t+\varphi) \\ v_{im} &= \mathrm{Im}(v) = \sqrt{2}V\sin(\omega t+\varphi) \end{aligned}\right\} \tag{2}$$

The current in single-phase networks is expressed as

$$i = \sqrt{2}Ie^{j(\omega t+\theta)} \tag{3}$$

where V is RMS value, θ is initial phase angle respectively. The real and imaginary components of the current are expressed as

$$\left.\begin{aligned} i_{re} &= \mathrm{Re}(i) = \sqrt{2}I\cos(\omega t+\theta) \\ i_{im} &= \mathrm{Im}(i) = \sqrt{2}I\sin(\omega t+\theta) \end{aligned}\right\} \tag{4}$$

We define instantaneous active and reactive power in single-phase networks as [2]

$$\left.\begin{aligned} p &= \frac{1}{2}\mathrm{Re}(vi+vi^*) \\ q &= \frac{1}{2}\mathrm{Im}(vi+vi^*) \end{aligned}\right\} \tag{5}$$

where * means conjugate complex number. We call vi as quadratic-frequency power and vi^* as differential-frequency power.

Substituting Equation (1) and (3) into Equation (5), we obtain

$$\left.\begin{aligned} P &= VI\cos(\varphi-\theta) + VI\cos(2\omega t+\varphi+\theta) \\ Q &= VI\sin(\varphi-\theta) + VI\sin(2\omega t+\varphi+\theta) \end{aligned}\right\} \tag{6}$$

Here the former part is a steady quantity that is differential-frequency power and the latter is oscillating in quadratic-frequency that is quadratic-frequency power.

Though we have definitions of instantaneous active and reactive power, it is difficult to measure them by Equation (5). Therefore we propose equivalent equations for measuring instantaneous active and reactive power next.

2.2 Measure Instantaneous Active and Reactive Power in Single-Phase Networks

Equivalent equations of instantaneous active and reactive power in single-phase networks are proposed as

$$\left.\begin{aligned} p &= v_{re} i_{re} = \frac{1}{2}\mathrm{Re}(vi + vi^*) \\ q &= v_{im} i_{re} = \frac{1}{2}\mathrm{Im}(vi + vi^*) \end{aligned}\right\} \tag{7}$$

where voltage imaginary component can be obtained as

$$\begin{aligned} v_{im} &= \sqrt{2}V\sin(\omega t + \varphi) = \sqrt{2}V\cos(\omega t + \varphi - \frac{\pi}{2}) \\ &= v_{re(t-T/4)} \end{aligned} \tag{8}$$

where T is period of AC power systems. With Equation (7), we can obtain instantaneous active and reactive power in single-phase networks theirs amplitudes are oscillating in quadratic frequency. It is widely considered that there is no instantaneous reactive power in single-phase networks. With above definitions, we can not only define instantaneous reactive power but also measure it in single-phase networks.

2.3 Define RMS Active and Reactive Power in Single-Phase Networks

RMS active power and reactive in single-phase networks are defined as

$$\left.\begin{aligned} P &= \frac{1}{T}\int_0^T p dt \\ Q &= \frac{1}{T}\int_0^T q dt \end{aligned}\right\} \tag{9}$$

Substituting Equation (5) into above equation, it can be proved that RMS power is equivalent to differential-frequency power in a steady state situation.

2.4 Measure RMS Active and Reactive Power in Single-Phase Networks

With Equation (7) and (9), integral method for measuring RMS active and reactive power in single-phase networks can be expressed as

$$\left.\begin{aligned} P &= \frac{1}{T}\int_0^T v_{re} i_{re} dt = \frac{1}{T}\sum_{k=1}^{T/\Delta t}(v_{re} i_{re})_k \Delta t \\ Q &= \frac{1}{T}\int_0^T v_{im} i_{re} dt = \frac{1}{T}\sum_{k=1}^{T/\Delta t}\{v_{re(t-T/4)} i_{re}\}_k \Delta t \end{aligned}\right\} \tag{10}$$

where Δt is the time step and it is more smaller the result is more precise.

Here we introduce another method to measure RMS reactive power in single-phase networks. First we measure RMS voltage and current in single-phase networks as

$$V = \sqrt{\frac{1}{T}\int_0^T v_{re}{}^2 dt} = \sqrt{\frac{1}{T}\sum_{k=1}^{T/\Delta t}(v_{re}{}^2)_k \Delta t} \tag{11}$$

$$I = \sqrt{\frac{1}{T}\int_0^T i_{re}{}^2 dt} = \sqrt{\frac{1}{T}\sum_{k=1}^{T/\Delta t}(i_{re}{}^2)_k \Delta t} \tag{12}$$

Then RMS reactive power can be obtained as

$$Q = \sqrt{(IV)^2 - P^2} \tag{13}$$

Because there is no phase angle shift operation, Equation (13) is considered more precise than Equation (10) in a transient state situation.

After obtained RMS active and reactive power, phase difference between voltage and current can calculated as

$$\delta = \varphi - \theta = \tan^{-1}(\frac{Q}{P}) \tag{14}$$

Equation (14) is very stable and very useful in control and protection systems.

3 MEASURE RMS ACTIVE AND REACTIVE POWER IN THREE-PHASE NETWORKS

This section develops integral method for measuring RMS active and reactive power in symmetrical and unsymmetrical three-phase networks.

3.1 Define RMS Active and Reactive Power in Symmetrical Three-phase Networks

The instantaneous voltages and currents in symmetrical three-phase networks can be expressed as

$$\begin{bmatrix} v_A \\ v_B \\ v_C \end{bmatrix} = \sqrt{2}Ve^{j(\omega t+\varphi)}\begin{bmatrix} 1 \\ e^{-j2\pi/3} \\ e^{j2\pi/3} \end{bmatrix} \tag{15}$$

$$\begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} = \sqrt{2}Ie^{j(\omega t+\varphi)}\begin{bmatrix} 1 \\ e^{-j2\pi/3} \\ e^{j2\pi/3} \end{bmatrix} \tag{16}$$

We can obtain instantaneous active and reactive power in symmetrical three-phase networks as

$$\begin{aligned} p_3 &= \frac{1}{2}\mathrm{Re}(v_A i_A + v_A i_A{}^* + \\ &\quad v_B i_B + v_B i_B{}^* + v_C i_C + v_C i_C{}^*) \\ &= 3VI\cos(\varphi - \theta) \end{aligned} \tag{17}$$

$$\begin{aligned} q_3 &= \frac{1}{2}\mathrm{Im}(v_A i_A + v_A i_A{}^* + \\ &\quad v_B i_B + v_B i_B{}^* + v_C i_C + v_C i_C{}^*) \\ &= 3VI\sin(\varphi - \theta) \end{aligned} \tag{18}$$

We can also obtain RMS active and reactive power in symmetrical three-phase networks according to the definitions as

$$P_3 = \frac{1}{T}\int_0^T p_3 dt = 3VI\cos(\varphi - \theta) = p_3 \tag{19}$$

$$Q_3 = \frac{1}{T}\int_0^T q_3 dt = 3VI\sin(\varphi - \theta) = q_3 \tag{20}$$

Compare Equations (19), (20) to Equations (17), (18), it can be found that RMS active and reactive power is equal to instantaneous active and reactive power in symmetrical three-phase networks respectively. Quadratic-frequency power that is existed in single-phase networks disappears in symmetrical three-phase networks.

3.2 Measure RMS Active and Reactive Power in Symmetrical Three-Phase Networks

With Equation (7), we simply obtain summation of RMS active and reactive power in symmetrical three-phase networks as

$$\left.\begin{aligned} P_3 &= \frac{1}{T}\int_0^T (v_{Are}i_{Are} + v_{Bre}i_{Bre} + v_{Cre}i_{Cre})dt \\ Q_3 &= \frac{1}{T}\int_0^T (v_{Aim}i_{Are} + v_{Bim}i_{Bre} + v_{Cim}i_{Cre})dt \end{aligned}\right\} \tag{21}$$

Integral method for measuring RMS active and reactive power in three-phase symmetrical networks can be expressed as

$$\left.\begin{aligned} P_3 &= \frac{1}{T}\sum_{k=1}^{T/\Delta t} (v_{Are}i_{Are} + v_{Bre}i_{Bre} + v_{Cre}i_{Cre})_k\, dt \\ Q_3 &= \frac{1}{T}\sum_{k=1}^{T/\Delta t} (v_{Aim}i_{Are} + v_{Bim}i_{Bre} + v_{Cim}i_{Cre})_k\, dt \end{aligned}\right\} \tag{22}$$

Here three-phase voltage imaginary components can be obtained as follows.

$$\left.\begin{aligned} v_{Aim} &= v_{Are(t-T/4)} \\ v_{Bim} &= v_{Bre(t-T/4)} \\ v_{Cim} &= v_{Cre(t-T/4)} \end{aligned}\right\} \tag{23}$$

However, we can also measure RMS voltages and currents, use Equation (13) to calculate RMS reactive power phase by phase in another way.

Though a power system is designed as symmetrical three-phase networks, there are many unsymmetrical three-phase phenomena because unbalanced loads and parameters of apparatus. Therefore it is very important to analyze unsymmetrical three-phase networks. Next, we will extend method of symmetrical coordinates from phasor state variables to instantaneous spiral vector state variables, then propose integral method for measuring RMS active power and reactive power of symmetrical phase sequence in unsymmetrical three-phase networks.

3.3 Define RMS Active and Reactive Power in Unsymmetrical Three-Phase Networks

Instantaneous voltage and current in unsymmetrical three-phase networks are expressed as

$$\left.\begin{aligned} v_A &= \sqrt{2}V_A e^{j(\omega t+\varphi_A)} \\ v_B &= \sqrt{2}V_B e^{j(\omega t+\varphi_B)} \\ v_C &= \sqrt{2}V_C e^{j(\omega t+\varphi_C)} \end{aligned}\right\} \tag{24}$$

$$\left.\begin{aligned} i_A &= \sqrt{2}I_A e^{j(\omega t+\theta_A)} \\ i_B &= \sqrt{2}I_B e^{j(\omega t+\theta_B)} \\ i_C &= \sqrt{2}I_C e^{j(\omega t+\theta_C)} \end{aligned}\right\} \tag{25}$$

where all state variables are instantaneous spiral vectors and they are different from phasors that are stationary complex numbers.

Though method of symmetrical coordinates is only applied to phasor state variables, we extend it to instantaneous spiral vector state variables.

The relations between instantaneous voltages of zero-phase sequence, positive-phase sequence, negative-phase sequence and instantaneous phase voltages in unsymmetrical three-phase networks are defined as

$$\begin{bmatrix} v_0 \\ v_1 \\ v_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} v_A \\ v_B \\ v_C \end{bmatrix} \tag{26}$$

where α is the unit vector and is defined as

$$\alpha = e^{j2\pi/3} = -\frac{1}{2} + j\frac{\sqrt{3}}{2} \tag{27}$$

The relations of currents are same defined as

$$\begin{bmatrix} i_0 \\ i_1 \\ i_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} \tag{28}$$

Assuming instantaneous voltages and currents of zero-phase sequence, positive-phase sequence, negative-phase sequence are expressed as

$$\left.\begin{aligned} v_0 &= \sqrt{2}V_0 e^{j(\omega t+\varphi_0)} \\ v_1 &= \sqrt{2}V_1 e^{j(\omega t+\varphi_1)} \\ v_2 &= \sqrt{2}V_2 e^{j(\omega t+\varphi_2)} \end{aligned}\right\} \tag{29}$$

$$\left.\begin{aligned} i_0 &= \sqrt{2}I_0 e^{j(\omega t+\theta_0)} \\ i_1 &= \sqrt{2}I_1 e^{j(\omega t+\theta_1)} \\ i_2 &= \sqrt{2}I_2 e^{j(\omega t+\theta_2)} \end{aligned}\right\} \tag{30}$$

We define instantaneous active and reactive power of zero-phase sequence as

$$\left.\begin{aligned} p_0 &= \frac{1}{2}\mathrm{Re}(v_0 i_0 + v_0 i_0^*) \\ q_0 &= \frac{1}{2}\mathrm{Im}(v_0 i_0 + v_0 i_0^*) \end{aligned}\right\} \tag{31}$$

RMS active and reactive power of zero-phase sequence can be obtained as

$$\left.\begin{aligned} P_0 &= \frac{1}{T}\int_0^T p_0 dt = V_0 I_0 \cos(\varphi_0 - \theta_0) \\ Q_0 &= \frac{1}{T}\int_0^T q_0 dt = V_0 I_0 \sin(\varphi_0 - \theta_0) \end{aligned}\right\} \tag{32}$$

We define instantaneous active and reactive power of positive-phase sequence as

$$\left.\begin{aligned} p_1 &= \frac{1}{2}\mathrm{Re}(v_1 i_1 + v_1 i_1^*) \\ q_1 &= \frac{1}{2}\mathrm{Im}(v_1 i_1 + v_1 i_1^*) \end{aligned}\right\} \tag{33}$$

RMS active and reactive power of positive-phase sequence can be obtained as

$$\left.\begin{aligned} P_1 &= \frac{1}{T}\int_0^T p_1 dt = V_1 I_1 \cos(\varphi_1 - \theta_1) \\ Q_1 &= \frac{1}{T}\int_0^T q_1 dt = V_1 I_1 \sin(\varphi_1 - \theta_1) \end{aligned}\right\} \tag{34}$$

We define instantaneous active and reactive power of negative-phase sequence as

$$\left.\begin{aligned} p_2 &= \frac{1}{2}\mathrm{Re}(v_2 i_2 + v_2 i_2^*) \\ q_2 &= \frac{1}{2}\mathrm{Im}(v_2 i_2 + v_2 i_2^*) \end{aligned}\right\} \tag{35}$$

RMS active and reactive power of negative-phase sequence can be obtained as

$$\left.\begin{aligned} P_2 &= \frac{1}{T}\int_0^T p_2 dt = V_2 I_2 \cos(\varphi_2 - \theta_2) \\ Q_2 &= \frac{1}{T}\int_0^T q_2 dt = V_2 I_2 \sin(\varphi_2 - \theta_2) \end{aligned}\right\} \tag{36}$$

After defined RMS active and reactive power of symmetrical phase sequence, we will show the relation between them and three phases RMS active and reactive power.

We have shown that in a steady state situation, RMS power is equal to differential-frequency power. Therefore, RMS apparent differential-frequency power in three-phase networks can be expressed as

$$S_3 = \frac{1}{2}(v_A i_A^* + v_B i_B^* + v_C i_C^*) \tag{37}$$

where its real component is active power and its imaginary component is reactive power. Substituting Equations (26), (28) into Equation (37), we obtain

$$\begin{aligned} S_3 &= \frac{1}{2}(v_A i_A^* + v_B i_B^* + v_C i_C^*) = \frac{1}{2}\begin{bmatrix} v_A & v_B & v_C \end{bmatrix}\begin{bmatrix} i_A^* \\ i_B^* \\ i_C^* \end{bmatrix} \\ &= \frac{1}{2}(3v_0 i_0^* + 3v_1 i_1^* + 3v_2 i_2^*) \end{aligned} \tag{38}$$

It can be found that no power happens among different phase sequence. From Equation (38), we obtain the relation equations between RMS power of symmetrical phase sequence and three phases.

$$\left.\begin{aligned} P_A + P_B + P_C &= 3(P_0 + P_1 + P_2) \\ Q_A + Q_B + Q_C &= 3(Q_0 + Q_1 + Q_2) \end{aligned}\right\} \tag{39}$$

According Equation (10), we can easily measure RMS active and reactive power phase by phase. Next we will propose integral method for measuring RMS active and reactive power of symmetrical phase sequence in unsymmetrical three-phase networks.

3.4 Measure RMS Active and Reactive Power in Unsymmetrical Three-Phase Networks

For measuring RMS active and reactive power of symmetrical phase sequence, instantaneous voltage and current of symmetrical phase sequence should be obtained firstly. For convenience, only current equations are shown.

Instantaneous current of zero-phase sequence can be obtained as

$$\left.\begin{aligned} i_{0re} &= \frac{1}{3}(i_{Are} + i_{Bre} + i_{Cre}) \\ i_{0im} &= \frac{1}{3}(i_{Aim} + i_{Bim} + i_{Cim}) \end{aligned}\right\} \tag{40}$$

Instantaneous current of positive-phase sequence can be obtained as

$$\left.\begin{aligned} i_{1re} &= \frac{1}{3}\{i_{Are} + i_{Bre(t-2T/3)} + i_{Cre(t-T/3)}\} \\ i_{1im} &= \frac{1}{3}\{i_{Aim} + i_{Bim(t-2T/3)} + i_{Cim(t-T/3)}\} \end{aligned}\right\} \tag{41}$$

Instantaneous current of negative-phase sequence can be obtained as

$$\left.\begin{aligned} i_{2re} &= \frac{1}{3}\{i_{Are} + i_{Bre(t-T/3)} + i_{Cre(t-2T/3)}\} \\ i_{2im} &= \frac{1}{3}\{i_{Aim} + i_{Bim(t-T/3)} + i_{Cim(t-2T/3)}\} \end{aligned}\right\} \tag{42}$$

We propose integral method for measuring RMS active and reactive power of zero-phase sequence as

$$\left.\begin{aligned} P_0 &= \frac{1}{T}\int_0^T p_0 dt = \frac{1}{T}\sum_{k=1}^{T/\Delta t}(v_{0re} i_{0re})_k \Delta t \\ Q_0 &= \frac{1}{T}\int_0^T q_0 dt = \frac{1}{T}\sum_{k=1}^{T/\Delta t}(v_{0im} i_{0re})_k \Delta t \end{aligned}\right\} \tag{43}$$

We propose integral method for measuring RMS active and reactive power of positive-phase sequence as

$$\left.\begin{aligned} P_1 &= \frac{1}{T}\int_0^T p_1 dt = \frac{1}{T}\sum_{k=1}^{T/\Delta t}(v_{1re} i_{1re})_k \Delta t \\ Q_1 &= \frac{1}{T}\int_0^T q_1 dt = \frac{1}{T}\sum_{k=1}^{T/\Delta t}(v_{1im} i_{1re})_k \Delta t \end{aligned}\right\} \tag{44}$$

We propose integral method for measuring RMS active and reactive power of negative-phase sequence as

$$\left.\begin{aligned} P_2 &= \frac{1}{T}\int_0^T p_2 dt = \frac{1}{T}\sum_{k=1}^{T/\Delta t}(v_{2re} i_{2re})_k \Delta t \\ Q_2 &= \frac{1}{T}\int_0^T q_2 dt = \frac{1}{T}\sum_{k=1}^{T/\Delta t}(v_{2im} i_{2re})_k \Delta t \end{aligned}\right\} \tag{45}$$

Because Equations (43)-(45) are more general than Equation (21), we recommend using them for measuring RMS active and reactive power in all three-phase networks. In most cases, compare to power summation of three phases, it is more important to know power of positive-phase sequence.

For analysis of unsymmetrical three-phase networks, it is also very important to measure RMS voltage and current of symmetrical phase sequence. With Equations (40)-(42), RMS current of symmetrical phase sequence can be easily obtained. In the same way, RMS voltage of symmetrical phase sequence can be obtained.

RMS voltage and current of zero-phase sequence can be obtained as

$$\left.\begin{aligned} V_0 &= \sqrt{\frac{1}{T}\int_0^T v_{0re}^2 dt} = \sqrt{\frac{1}{T}\sum_{k=1}^{T/\Delta t}(v_{0re}^2)_k \Delta t} \\ I_0 &= \sqrt{\frac{1}{T}\int_0^T i_{0re}^2 dt} = \sqrt{\frac{1}{T}\sum_{k=1}^{T/\Delta t}(i_{0re}^2)_k \Delta t} \end{aligned}\right\} \tag{46}$$

RMS voltage and current of positive-phase sequence can be obtained as

$$\left.\begin{aligned} V_1 &= \sqrt{\frac{1}{T}\int_0^T {v_{1re}}^2 dt} = \sqrt{\frac{1}{T}\sum_{k=1}^{T/\Delta t}({v_{1re}}^2)_k \Delta t} \\ I_1 &= \sqrt{\frac{1}{T}\int_0^T {i_{1re}}^2 dt} = \sqrt{\frac{1}{T}\sum_{k=1}^{T/\Delta t}({i_{1re}}^2)_k \Delta t} \end{aligned}\right\} \tag{47}$$

RMS voltage and current of negative-phase sequence can be obtained as

$$\left.\begin{aligned} V_2 &= \sqrt{\frac{1}{T}\int_0^T {v_{2re}}^2 dt} = \sqrt{\frac{1}{T}\sum_{k=1}^{T/\Delta t}({v_{2re}}^2)_k \Delta t} \\ I_2 &= \sqrt{\frac{1}{T}\int_0^T {i_{2re}}^2 dt} = \sqrt{\frac{1}{T}\sum_{k=1}^{T/\Delta t}({i_{2re}}^2)_k \Delta t} \end{aligned}\right\} \tag{48}$$

After obtained RMS voltages and currents in unsymmetrical three-phase, we can also get RMS reactive power according to Equation (13) in another way.

4 A NUMERICAL EXAMPLE

We will calculate an unsymmetrical three-phase circuit shown in Figure 1.

The general equation of the circuit is

$$\begin{bmatrix} v_A \\ v_B \\ v_C \end{bmatrix} = \begin{bmatrix} i_A Z_A \\ i_B Z_B \\ i_C Z_C \end{bmatrix} + \begin{bmatrix} i_N Z_N \\ i_N Z_N \\ i_N Z_N \end{bmatrix} \tag{49}$$

We transform three phases impedances into symmetrical phase sequence as

$$\begin{bmatrix} Z_0 \\ Z_1 \\ Z_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} Z_A \\ Z_B \\ Z_C \end{bmatrix} \tag{50}$$

Substituting Equation (50) and Equations (29), (30) into Equation (49), we obtain current solutions as

$$\begin{bmatrix} i_0 \\ i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} Z_0 + 3Z_N & Z_2 & Z_1 \\ Z_1 & Z_0 & Z_2 \\ Z_2 & Z_1 & Z_0 \end{bmatrix}^{-1}\begin{bmatrix} v_0 \\ v_1 \\ v_2 \end{bmatrix} \tag{51}$$

With equations shown in section 3, we can obtain numerical results of the circuit. Here we show some graphs. The parameters of the circuit is shown in Table 1.

In Figure 3, it can be found that current of positive-phase sequence is opposite with current of negative-phase sequence in phase angle.

In Figure 4 and Figure 5, instantaneous apparent power is rotating quadratic-frequency in the complex plane its real axis represents active power and its imaginary axis represents reactive respectively. The trajectory of apparent power is a circle and its center is the coordinate point of RMS power.

From Figure 6 to Figure 9, comparison of calculated and measured RMS power is shown. Here calculated power means power calculated from theory equation (like Equation (32)) and measured power means power obtained from integral method equations (like Equation (43)). All of them agree with each other very well.

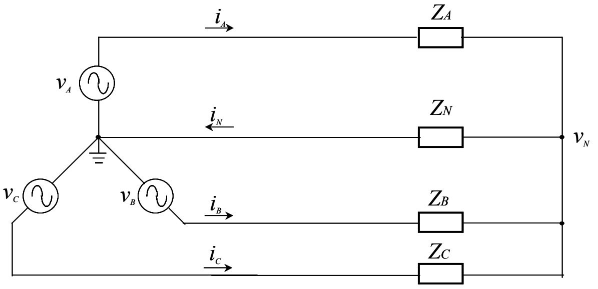

Figure 1 An unsymmetrical three-phase circuit

Table 1 Parameters of the unsymmetrical three-phase circuit

f=50(Hz)
$V_A=1(V)$, $\varphi_A=0(rad)$
$V_B=3(V)$, $\varphi_B=-1(rad)$
$V_C=2(V)$, $\varphi_C=2(rad)$
$Z_A=1+j2(\Omega)$
$Z_B=2+j5(\Omega)$
$Z_C=3+j8(\Omega)$
$Z_N=1+j1(\Omega)$

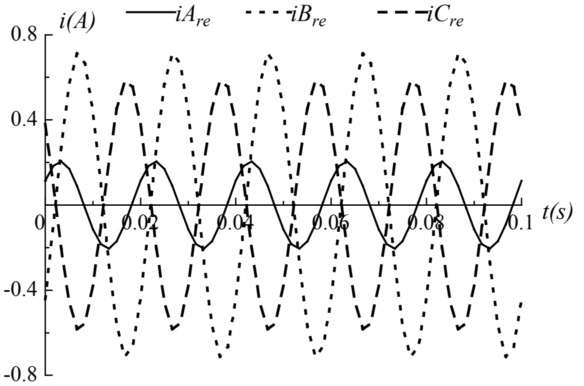

Figure 2 Instantaneous phase current real values

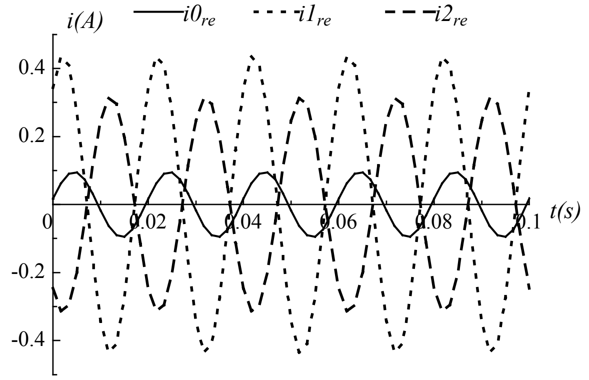

Figure 3 Instantaneous current real values of symmetrical phase sequence

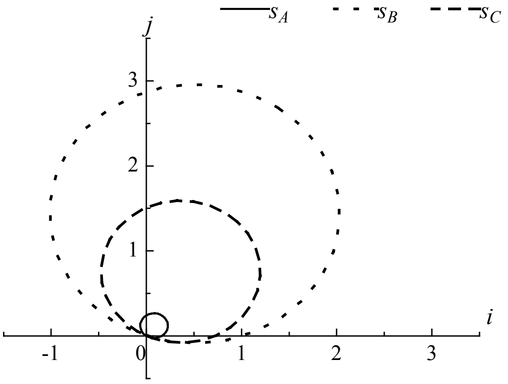

Figure 4 Instantaneous apparent power in the complex plane

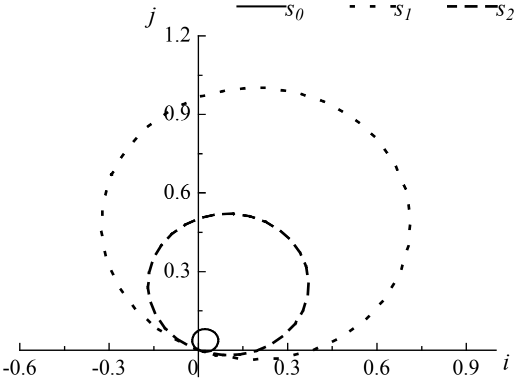

Figure 5 Instantaneous apparent power of symmetrical phase sequence in the complex plane

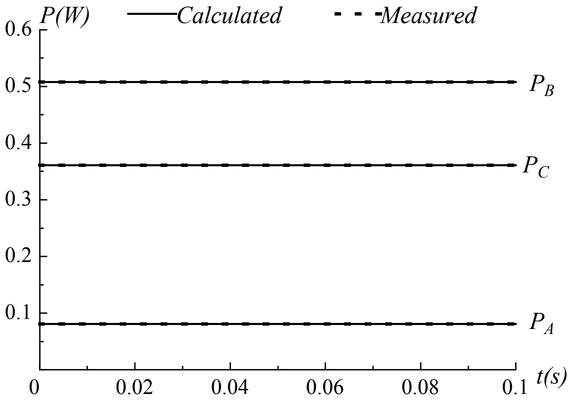

Figure 6 Comparison of calculated and measured RMS active power

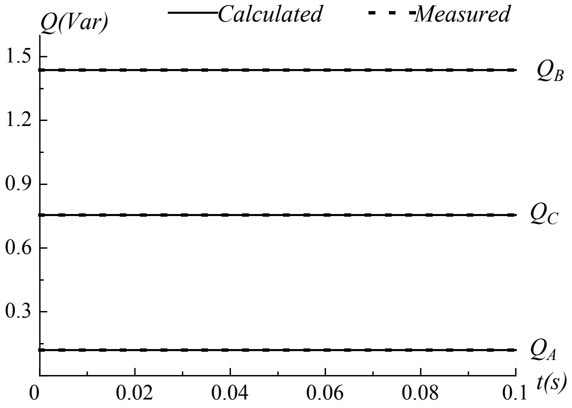

Figure 7 Comparison of calculated and measured RMS reactive power

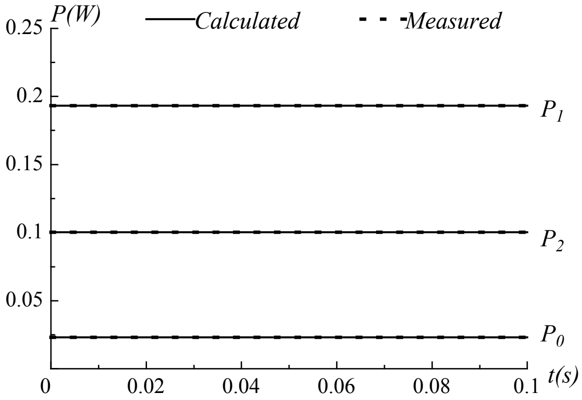

Figure 8 Comparison of calculated and measured RMS active power of symmetrical phase sequence

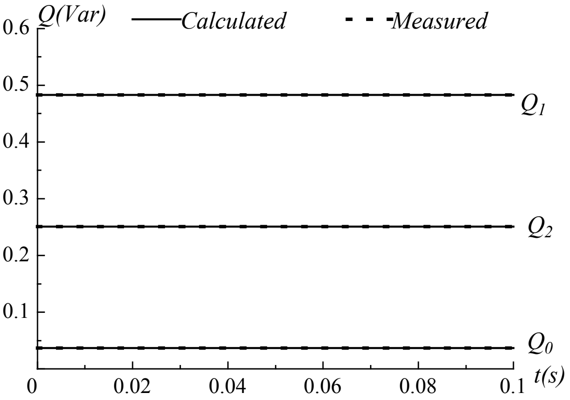

Figure 9 Comparison of calculated and measured RMS reactive power of symmetrical phase sequence

5 CONCLUSION

The paper developed an integral method for measuring RMS active and reactive power in single- and multiphase networks according to spiral vector theory. The originality of the paper is that it defined instantaneous reactive power in single-phase networks that is considered not existed widely. The numerical example shown that the new method can effective measure RMS symmetrical phase sequence active and reactive power in unsymmetrical three-phase networks that are very important for control and analysis of power systems.

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