Strategic Obscurity in the Forecasting of Disasters*

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Abstract

A principal acquires information about a shock and then discloses it to an agent. After the disclosure, the principal and agent each decide whether to take costly preparatory actions that yield mutual benefits but only when the shock strikes. The principal maximizes his expected payoff by controlling the quality of his information, and the disclosure rule. We show that even when the acquisition of perfect information is costless, the principal may optimally acquire imperfect information when his own action eliminates the agent's incentive to take action against the risk.

Key words: endogenous information, disclosure, signal quality, transparency, specific investment, strategic ignorance.
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1 Introduction

Preparing for a variety of natural, social, and economic shocks is an important task of every government. Many governments appropriate a large amount of money on research into the forecasting of such natural shocks as hurricanes, snow storms and other extreme weather conditions, earthquakes, epidemic outbreaks, and so on.[1]

Along with forecasting, a government's strategies to prepare for those shocks typically involve two forms of interventions. The first is a direct intervention that is implemented at the government's own cost. The second is an indirect intervention that consists of raising public awareness of the risk of the shocks and advising the public to take preparatory actions themselves. In the case of an epidemic outbreak, for example, the direct interventions include stricter quarantine control, building depressurized rooms at hospitals, increasing the stock of anti-virus medicines, and so on. On the other hand, an indirect intervention consists of advice to the public to receive vaccinations, avoid traveling and exercise hygiene practices. Likewise, against earthquakes, direct interventions include enforcing stricter building codes and reinforcing public buildings such as schools and highways, while indirect interventions include advice to the public to reinforce their own houses, prepare food stocks, and purchase earthquake insurance. Unlike direct interventions, it is the public themselves who bear the cost of the advised action.[2] The essential feature of many of these preparatory actions is that they are *specific investment* in the sense that they have value only when the shock strikes.

It is argued by some that the policy of spending much money on forecasting shocks and at the same time advising the public to take preparatory measures is inconsistent.[3] One interpretation of this claim is as follows: If the accurate forecasting of a shock is possible, then the public is led to think that timely direct

[1]For example, National Oceanic and Atmospheric Agency (NOAA) of the United States budgeted more than $2,000 million on weather services and satellites. Its joint polar satellite system (JPSS), which is used for mid-range weather forecasts, alone cost US$382 million in FY2010 ("NOAA warns weather forecasts will suffer from budget cuts," Washington Post 03/31/2011). As another example, the US Geological Survey budgeted more than US$90 million for research into geologic hazard assessments in FY2010.

[2]Skoufias (2003) discusses the strategies employed by households and public agencies to mitigate the damages of economic crises and natural disasters. Some indirect interventions involve public expenditure as in the case of subsidies for vaccination programs, or those for the installation of solar panels.

[3]See Saito (2008).

interventions will save them costly efforts. On the other hand, from the point of view of the government, indirect interventions are much less costly and the public's own action is often more effective in mitigating the damage.

The purpose of this paper is to provide a formal examination of the above logic in a stylized model where a principal (government) acquires information and then discloses it to an agent (the public). We show that acquiring perfect information may indeed be suboptimal for the principal when the agent can free-ride on his effort. A more detailed description of the model is as follows: Facing the risk of a shock, the principal first chooses the technology that determines the quality of his private information about the risk of the shock. The technology r can be any real number between 0 and ∞, where $r = 0$ corresponds to perfect information, and $r > 0$ corresponds to information with noise. Choice of any technology is costless. Upon acquiring information, the principal determines whether to take a preparatory action, and at the same time advises the agent on whether he should take a costly preparatory action. The preparatory actions yield mutual benefits, but only when the shock strikes. We specify the payoffs in the shock state as follows: For the principal, taking action is a dominant strategy. That is, when the shock occurs for sure, the principal cannot commit to not taking action. On the other hand, the agent has a free-ride incentive in the sense that taking action is optimal if and only if the principal does not.

We first show that when the prior probability of the shock is moderately high, acquiring no information is better for the principal than acquiring perfect information. When the prior probability is low, however, no information is dominated by perfect information. This leads us to the question on whether there still exists an imperfect information policy that outperforms perfect information even for low probability shocks. For this, we suppose that the signal space is continuous and consider an imperfect information policy that works according to the three risk categories as follows: When the updated risk is high based on the acquired information, the principal takes action but recommends *no* action to the agent. When the risk is medium, the principal takes no action but recommends an action to the agent. When the risk is low, the principal takes no action and also recommends no action to the agent. With appropriate choice of the thresholds, we note that this policy is equivalent to the full disclosure of private information. Our main result shows that such a full disclosure policy with imperfect information dominates perfect information when the marginal benefit of the agent's action is sufficiently large for the principal, or when the agent's utility from inaction by both parties is sufficiently low in the event of the shock. These conditions are hence relevant when the public has significantly more efficient ways to insure against the risk, or when the shock has a disastrous consequence when no preparation is made. We further present a characterization of the optimal (partial) disclosure rule for a given information quality.

The paper is organized as follows: After the discussion of the related literature in the next section, we formulate a model of information acquisition and disclosure in Section 3, and present some preliminary analysis in Section 4. Section 5 compares the two extreme cases of perfect information and no information. Optimality of imperfect information is illustrated in Section 6 using a simple model with finite signals. Section 7 presents the main theorem establishing the optimality of imperfect information with continuous signals. A characterization of an optimal disclosure rule for a given signal quality is provided in Section 8. We conclude in Section 9. All the proofs are collected in the Appendix.

2 Related Literature

Decision making in the face of a natural shock is a classical subject in both the theoretical and empirical literature. Nelson and Winter (1964) study the weather forecasting system that maximizes the welfare of its user who must decide whether to take a protective action against rain. Howe and Cochrane (1974) study the decision problem faced by authorities under a snow storm forecast. Their empirical observation on the "reluctance on the part of snow removal authorities to be sensitive to any but very severe forecasts in making operation decisions" is consistent with the optimal policy in the current paper. Brookshire *et al.* (1985) show that the expected utility hypothesis is a reasonable description of decision-making behavior facing a low-probability, high-loss event of an earthquake. Lewis and Nickerson (1989) study the interaction of self-insurance and public interventions against natural disasters.

Information acquisition and disclosure is an increasingly popular topic in the theoretical literature. Combination of the following elements is a distinguishing feature of the present model and has not been studied together to the best of our knowledge.

- The principal engages in information acquisition and information disclosure.[4]

[4]Matthews and Postlewaite (1985) study a model of sales where a seller tests the quality of his good and then discloses it to a buyer.

- Information acquisition is costless.
- The principal has a continuous choice of information quality.[5]

Principal-agent models of information acquisition in the literature are divided into two groups depending on who acquires information.[6] Cremer *et al.* (1998a, b), Kessler (1998), Lewis and Sappington (1993, 1997), Szalay (2005, 2009), and Dai *et al.* (2006) study the design of an optimal contract when an agent can privately invest resources to acquire information either before or after the contract is signed. In these models, positive cost of information acquisition is a critical element that determines the form of an optimal contract as well as the agent's decision to become informed. The second class of models assume information acquisition by the principal and examine whether ignorance helps the principal commit to some decision in a subsequent interaction with the agent. Among others, Dewatripont and Maskin (1995) show that simple contracts based on the limited observation of variables may be superior to more complete contracts when renegotiation is possible, and Cremer (1995) shows that the principal may choose to acquire no information about the agent's productivity in a dynamic model with adverse selection.[7] Like these models, we assume that the principal acquires information and then plays a game against the agent.[8] We show that even when complete ignorance cannot serve as a commitment device, a variable degree of incomplete ignorance (*i.e.,* acquisition of imperfect information) may still be a useful commitment device.

The choice of signal quality in information disclosure problems is studied by Lewis and Sappington (1994) and Bergemann and Pesendorfer (2007), who both analyze a seller's problem when he chooses the quality of buyers' private signals. In these models, hence, the player who controls the signal quality does not observe the resulting information. Kamenica and Gentzkow (2011) study information acquisition by a sender when the signal is publicly observable as in the case of full disclosure in our model. When the sender has no action to take, they ask whether or not acquisition of some information dominates *no* information. In contrast, our

[5]Szalay (2009) analyzes the continuous choice of information quality.

[6]Information acquisition is also studied in a more abstract mechanism design setting as well as in auctions.

[7]Carrillo and Mariotti (2000) demonstrate strategic ignorance by a decision maker who has time-inconsistent preferences.

[8]Lack of commitment by a mechanism designer is studied by Bester and Strausz (2000), and Skreta (2006). Note that solicitation of agents' private information is absent in our model.

focus is on the comparison between the acquisition of imperfect information and that of perfect information when the sender of information also has an action to take.

Finally, it is also possible to relate our finding to the literature on government transparency, which asks whether disclosure of a government's private information induces inefficient coordination by the public and creates uninsurable risks. Our conclusion points to the possibility that even full disclosure takes place, the content of information may be less than what is potentially available if information acquisition is endogenous.[9]

3 Model

There are a principal (player 1) and an agent (player 2) facing the risk of a shock. The shock corresponds to one of the two states of the world $\omega \in \Omega$: The shock occurs in state $\omega = 1$ and does not in state $\omega = 0$. The prior probability of the shock equals $p = P(\omega = 1) \in (0, 1)$. Before the state is realized, each player i either "takes action" ($a_i = 1$) or not ($a_i = 0$) against the shock. Denote by $A_i = \{0, 1\}$ the set of actions of player i. We suppose that actions are taken simultaneously after the disclosure. The players' payoffs depend on the action profile and the state. Specifically, player i's payoff under the action profile $a = (a_1, a_2)$ in state ω is given by

$$v_i(a, \omega) = u_i(a)\omega - c_i a_i.$$

Hence, the players benefit from the actions only when there is a shock ($\omega = 1$), but incur the cost c_i of taking action even when there is no shock. Let

$$\begin{aligned}
d_1^0 &= u_1(1,0) - u_1(0,0), \quad & d_1^1 &= u_1(1,1) - u_1(0,1), \\
d_2^0 &= u_2(0,1) - u_2(0,0), \quad & d_2^1 &= u_2(1,1) - u_2(1,0), \\
m_1^0 &= u_1(0,1) - u_1(0,0), \quad & m_1^1 &= u_1(1,1) - u_1(1,0).
\end{aligned}$$

d_1^0 is the marginal benefit of his own action $a_1 = 1$ to the principal when it is unilaterally taken, and d_1^1 is the marginal benefit of $a_1 = 1$ when the agent also chooses $a_2 = 1$. d_2^0 and d_2^1 are the corresponding quantities for the agent. m_1^0 and m_1^1 are the marginal benefits of agent's action to the principal when the principal

[9] The literature originates with Hirschleifer (1971), and subsequent developments include Morris and Shin (2002), Svensson (2006) and Walsh (2007).

himself chooses $a_1 = 0$ and $a_1 = 1$, respectively. We assume that

$$d_1^0 \geq d_1^1 > c_1 > 0, \tag{1}$$

$$d_2^0 > c_2 > d_2^1 > 0, \tag{2}$$

$$m_1^0 > d_1^0 - c_1. \tag{3}$$

(1) and (2) show that the two players' actions are strategic substitutes: The marginal benefit of the own action is higher when it is unilateral. Furthermore, (1) says that $a_1 = 1$ is a dominant action for the principal in the event of a sure shock, and (2) says that the agent's best response is to take action when the principal does not, and vice versa. (3) says that for the principal, the marginal benefit of the agent's unilateral action is higher than the net marginal benefit of his own unilateral action. For concreteness, we also assume in what follows that

$$\frac{d_2^0}{c_2} > \frac{d_1^0}{c_1}. \tag{4}$$

In other words, when normalized by its cost, the agent's unilateral action raises his own utility more than the principal's unilateral action raises his.

When the principal chooses to acquire information, he observes signal θ in set $\Theta \subset \boldsymbol{R}$, which we assume to be either finite or continuous in our discussion. His *forecasting technology* r determines the level of accuracy of θ in a sense made precise below. Depending on whether Θ is finite or continuous, let $f_{\omega,r}(\theta)$ denote either the probability or density of signal θ in state ω when the forecasting technology is r. Under any technology, the higher the signal θ, the more likely is $\omega = 1$ as expressed by the *monotone likelihood ratio property* below:

$$\theta < \theta' \quad \Rightarrow \quad \frac{f_{0,r}(\theta)}{f_{1,r}(\theta)} > \frac{f_{0,r}(\theta')}{f_{1,r}(\theta')}. \tag{5}$$

The timing of events is as follows. First, the principal chooses his forecasting technology r. Upon observing θ, he makes a non-binding advice to the agent on which action to take. Both parties then choose actions simultaneously. The principal's action choice is based on his signal θ, while the agent's action choice is based on the principal's advice. Finally, the state is realized and the players receive payoffs.

The principal's choice of an advice given the observation of θ is expressed by a *disclosure rule* $g : \Theta \to \{0, 1\}$: $g(\theta)$ is the action suggested to the agent when θ is observed. The principal's *policy* is a pair (r, g) of his forecasting technology and disclosure rule.[10] We assume that the policy (r, g) is chosen in advance and is publicly announced. Public observability of the forecasting technology r is a reasonable assumption given that it usually entails publicly observable activities such as launching a satellite, building a supercomputer or a network of sensors, and so on. We also assume that the principal commits to his disclosure rule g in the sense that for any signal θ, his advice equals $g(\theta)$.[11]

Given a policy (r, g), each player's strategy is defined as follows. The principal's strategy $\sigma_1 : \boldsymbol{R} \to \{0, 1\}$ chooses an action as a function of the observed signal θ. On the other hand, the agent's strategy $\sigma_2 : \{0, 1\} \to \{0, 1\}$ chooses an action as a function of the principal's advice. Let σ_2^* denote the *obedient* strategy such that $\sigma_2^*(a_2) = a_2$ for any $a_2 \in \{0, 1\}$. Let $\pi_i(\sigma \mid r, g)$ denote player i's ex ante expected payoff under the strategy profile $\sigma = (\sigma_1, \sigma_2)$ and the policy (r, g). Explicitly, they are given by

$$\begin{aligned} \pi_1(\sigma \mid r, g) &= E_{\omega,\theta}\Big[u_1\left(\sigma_1(\theta), \sigma_2(g(\theta))\right)\omega - c_1\sigma_1(\theta)\Big], \\ \pi_2(\sigma \mid r, g) &= E_{\omega,\theta}\Big[u_2\left(\sigma_1(\theta), \sigma_2(g(\theta))\right)\omega - c_2\sigma_2(g(\theta))\Big]. \end{aligned} \tag{6}$$

The strategy profile σ is a (Bayes-Nash) *equilibrium under* (r, g) if $\pi_i(\sigma \mid r, g) \geq \pi_i(\sigma_i', \sigma_j \mid r, g)$ for any σ_i' and $i \neq j$.[12] A policy (r, g) is *incentive compatible* if there exists a strategy σ_1 of the principal such that (σ_1, σ_2^*) is an equilibrium under (r, g). For an incentive compatible policy (r, g), if σ_1 is understood, we simply write $\pi_i(r, g)$ for the equilibrium payoff $\pi_i(\sigma_1, \sigma_2^* \mid r, g)$.

An incentive compatible policy (r, g) is *optimal* if there exists no other incentive compatible policy that yields a strictly higher equilibrium payoff. In other words, (r, g) is optimal if there exists σ_1 such that $\sigma = (\sigma_1, \sigma_2^*)$ is an equilibrium under (r, g), and for any policy (r', g') under which $\sigma' = (\sigma_1', \sigma_2^*)$ is an equilibrium for

[10] An alternative definition of a disclosure rule is to specify the message space Y along with the mapping $g : \Theta \to Y$. For example, (g, Y) such that $Y = \Theta$ and $g(\theta) = \theta$ corresponds to full disclosure, (g, Y) such that $Y = \{0\}$ corresponds to no disclosure, *etc.* By the generalized revelation principle of Myerson (1982), however, no generality is lost when we assume that $Y = \{0, 1\}$ and hence that g generates an advice to the agent: Any message $y \in Y$ is associated in equilibrium with one of two actions of the agent, and the principal can suggest this action instead of y. Use of random disclosure rules does not change the conclusions of Sections 7 and 8.

[11] This is a standard assumption in the information revelation literature, and is most likely justified for disclosure by a public sector, where adherence to the publicly announced rule is verifiable through official documents.

[12] Use of a stronger notion of equilibrium does not affect the conclusions of the paper.

some σ_1', we have

$$\pi_1(\sigma \mid r, g) \geq \pi_1(\sigma' \mid r', g').$$

Let

$$\begin{aligned}
\delta_1^0 &= \tfrac{d_1^0}{c_1} - 1, & \delta_1^1 &= \tfrac{d_1^1}{c_1} - 1, \\
\delta_2^0 &= \tfrac{d_2^0}{c_2} - 1, & \delta_2^1 &= \tfrac{d_2^1}{c_2} - 1, \\
\mu_1^0 &= \tfrac{m_1^0}{c_1}, & \mu_1^1 &= \tfrac{m_1^1}{c_1}.
\end{aligned}$$

δ_1^0 is the net marginal benefit for the principal of his own action normalized by its cost when the agent does not take action, and δ_1^1 is the corresponding quantity when the agent takes action. δ_2^0 and δ_2^1 have similar interpretations for the agent. μ_1^0 is the normalized marginal benefit for the principal of the agent's action.

4 Preliminary Analysis

It is instructive to consider first the equilibrium action profile when the principal's signal θ is publicly observable. Let the technology r be given. When the signal is θ, the principal chooses $a_1 = 1$ if

$$E_\omega \left[u_1(1, \sigma_2(\theta))\omega \mid \theta \right] - c_1 > E_\omega \left[u_1(0, \sigma_2(\theta))\omega \mid \theta \right].$$

Upon simplification, we see that this is equivalent to

$$\begin{aligned}
\frac{f_{0,r}(\theta)}{f_{1,r}(\theta)} &< \frac{p\delta_1^1}{1-p} \text{ if } \sigma_2(\theta) = 1, \\
\frac{f_{0,r}(\theta)}{f_{1,r}(\theta)} &< \frac{p\delta_1^0}{1-p} \text{ if } \sigma_2(\theta) = 0.
\end{aligned}$$

Since $\delta_1^1 \leq \delta_1^0$ by assumption, it follows that $a_1 = 1$ is a dominant action for the principal if the likelihood ratio $\frac{f_{0,r}(\theta)}{f_{1,r}(\theta)} < \frac{p\delta_1^1}{1-p}$. Conversely, $a_1 = 0$ is a dominant action for the principal if the ratio is $> \frac{p\delta_1^0}{1-p}$. For later reference, we define these marginal values of θ by β^0 and β as follows:

$$\frac{f_{0,r}(\beta)}{f_{1,r}(\beta)} = \frac{p\delta_1^1}{1-p} \quad \text{and} \quad \frac{f_{0,r}(\beta^0)}{f_{1,r}(\beta^0)} = \frac{p\delta_1^0}{1-p}. \tag{7}$$

Since the likelihood ratio is strictly decreasing by (5), the principal has a dominant action $a_1 = 1$ if $\theta > \beta$ and $a_1 = 0$ if $\theta < \beta^0$. As for the agent, he chooses $a_2 = 1$ if

$$E_{\omega,\theta} \left[u_2(\sigma_1(\theta), 1)\omega \mid \theta \right] - c_2 > E_{\omega,\theta} \left[u_2(\sigma_1(\theta), 0)\omega \mid \theta \right],$$

or equivalently, if

$$\frac{f_{0,r}(\theta)}{f_{1,r}(\theta)} < \frac{p\delta_2^1}{1-p} \text{ if } \sigma_1(\theta) = 1,$$
$$\frac{f_{0,r}(\theta)}{f_{1,r}(\theta)} < \frac{p\delta_2^0}{1-p} \text{ if } \sigma_1(\theta) = 0.$$

Since $\delta_2^1 < 0$ by assumption, the agent never takes $a_2 = 1$ when $a_1 = 1$. Let α_2^0 be the marginal value of θ at which he is indifferent between $a_2 = 0$ and $a_2 = 1$:

$$\frac{f_{0,r}(\alpha^0)}{f_{1,r}(\alpha^0)} = \frac{p\delta_2^0}{1-p}. \tag{8}$$

That is, when the principal chooses $a_1 = 0$, the agent chooses $a_2 = 0$ if $\theta < \alpha^0$ and $a_2 = 1$ if $\theta > \alpha^0$. Noting that $\alpha^0 < \beta^0 < \beta$, we can summarize the equilibrium action profile when θ is publicly observable as follows:

$$(\sigma_1(\theta), \sigma_2(\theta)) = \begin{cases} (1,0) & \text{if } \frac{f_{0,r}(\theta)}{f_{1,r}(\theta)} < \frac{p\delta_1^1}{1-p} & \Leftrightarrow \theta > \beta, \\ (1,0) \text{ or } (0,1) & \text{if } \frac{f_{0,r}(\theta)}{f_{1,r}(\theta)} \in \left(\frac{p\delta_1^1}{1-p}, \frac{p\delta_1^0}{1-p}\right) & \Leftrightarrow \theta \in \left(\beta^0, \beta\right), \\ (0,1) & \text{if } \frac{f_{0,r}(\theta)}{f_{1,r}(\theta)} \in \left(\frac{p\delta_1^0}{1-p}, \frac{p\delta_2^0}{1-p}\right) & \Leftrightarrow \theta \in \left(\alpha^0, \beta^0\right), \\ (0,0) & \text{if } \frac{f_{0,r}(\theta)}{f_{1,r}(\theta)} > \frac{p\delta_2^0}{1-p} & \Leftrightarrow \theta < \alpha^0. \end{cases} \tag{9}$$

This exercise shows that the likelihood ratio of the two states given the signal θ is what determines the equilibrium behavior at θ.

Suppose next that θ is not publicly observed so that the agent must infer the realization of θ through the advice given by the principal. Let a policy (r, g) and the principal's strategy σ_1 be given. It is first clear that if (r, g) is incentive compatible with an equilibrium (σ_1, σ_2^*), then the principal's action $\sigma_1(\theta)$ at θ is a best response to his advice $g(\theta)$ to the agent. To see next the agent's incentive to follow the principal's advice, define

$$A_0 = \{\theta : \sigma_1(\theta) = 0\}, \quad A_1 = \{\theta : \sigma_1(\theta) = 1\},$$
$$B_0 = \{\theta : g(\theta) = 0\}, \quad B_1 = \{\theta : g(\theta) = 1\}.$$

Given the action-advice pair (σ, g), A_0 is the set of signals at which the principal chooses no action, and A_1 is the set of signals at which he takes action. B_0 and B_1 have similar interpretations. By (6), the agent follows the advice $a_2 = 1$ if

$$E_{\omega,\theta}\left[u_2(0,1)\omega\mathbf{1}_{\{\theta\in A_0\}} + u_2(1,1)\omega\mathbf{1}_{\{\theta\in A_1\}} \mid \theta \in B_1\right] - c_2$$
$$\geq E_{\omega,\theta}\left[u_2(0,0)\omega\mathbf{1}_{\{\theta\in A_0\}} + u_2(0,1)\omega\mathbf{1}_{\{\theta\in A_1\}} \mid \theta \in B_1\right],$$

and he follows the advice $a_2 = 0$ if

$$\begin{aligned}
&E_{\omega,\theta}\left[u_2(0,1)\omega\mathbf{1}_{\{\theta\in A_0\}} + u_2(1,1)\omega\mathbf{1}_{\{\theta\in A_1\}} \mid \theta \in B_0\right] - c_2 \\
&\leq E_{\omega,\theta}\left[u_2(0,0)\omega\mathbf{1}_{\{\theta\in A_0\}} + u_2(0,1)\omega\mathbf{1}_{\{\theta\in A_1\}} \mid \theta \in B_0\right],
\end{aligned}$$

where $\mathbf{1}_Z$ is an indicator function of event Z. Simple algebra shows that these inequalities are equivalent to (10) and (11) below.

$$\begin{aligned}
&\frac{p}{1-p}\left[P(\theta \in A_0 \cap B_1 \mid \omega = 1)\delta_2^0 + P(\theta \in A_1 \cap B_1 \mid \omega = 1)\delta_2^1\right] \\
&\qquad \geq P(\theta \in B_1 \mid \omega = 0),
\end{aligned} \tag{10}$$

and

$$\begin{aligned}
&\frac{p}{1-p}\left[P(\theta \in A_0 \cap B_0 \mid \omega = 1)\delta_2^0 + P(\theta \in A_1 \cap B_0 \mid \omega = 1)\delta_2^1\right] \\
&\qquad \leq P(\theta \in B_0 \mid \omega = 0).
\end{aligned} \tag{11}$$

5 Perfect Information and No Information

We now turn to the analysis of perfect information and no information policies. In the no information case, the equilibrium is determined by the prior probability alone. Since we can identify $\frac{f_{0,r}(\theta)}{f_{1,r}(\theta)} = 1$ under no information, (9) implies that the equilibrium action is given by

$$(\sigma_1, \sigma_2) = \begin{cases} (1,0) & \text{if } \frac{p\delta_1^1}{1-p} > 1 \qquad \Leftrightarrow p > \frac{c_1}{d_1^1}, \\ (1,0) \text{ or } (0,1) & \text{if } \frac{p\delta_1^1}{1-p} < 1 < \frac{p\delta_1^0}{1-p} \Leftrightarrow p \in \left(\frac{c_1}{d_1^0}, \frac{c_1}{d_1^1}\right), \\ (0,1) & \text{if } \frac{p\delta_1^1}{1-p} < 1 < \frac{p\delta_2^0}{1-p} \Leftrightarrow p \in \left(\frac{c_2}{d_2^0}, \frac{c_1}{d_1^0}\right), \\ (0,0) & \text{if } \frac{p\delta_2^0}{1-p} < 1 \qquad \Leftrightarrow p < \frac{c_2}{d_2^0}. \end{cases}$$

Since $u_1(1,0) - c_1 < u_1(0,1)$ by (3), the principal is better off with $(a_1, a_2) = (0,1)$ than with $(a_1, a_2) = (1,0)$. It follows that the optimal policy in the second case above should have $(a_1, a_2) = (0,1)$. Hence, the action-advice pair under the optimal no information policy is given by

$$(\sigma_1, g) = \begin{cases} (0,0) & \text{if } p < \frac{c_2}{d_2^0} \\ (0,1) & \text{if } \frac{c_2}{d_2^0} \leq p < \frac{c_1}{d_1^1}, \\ (1,0) & \text{if } p \geq \frac{c_1}{d_1^1}, \end{cases}$$

and the principal's payoff is given by

$$\pi_1(\sigma \mid r, g) = \begin{cases} pu_1(0,0) & \text{if } p < \frac{c_2}{d_2^0}. \\ pu_1(0,1) & \text{if } \frac{c_2}{d_2^0} \leq p < \frac{c_1}{d_1^1}, \\ pu_1(1,0) - c_1 & \text{if } p \geq \frac{c_1}{d_1^1}. \end{cases} \tag{12}$$

Next, in the case of perfect information, $\theta = \omega$, and the likelihood ratio $\frac{f_{r,0}(\omega)}{f_{1,r}(\omega)} = 0$ in state $\omega = 1$ and $= \infty$ in state $\omega = 0$. Note from (9) that in neither case, the agent chooses $a_2 = 1$ in equilibrium. It follows that an incentive compatible policy must advise no action whether $\theta = 0$ or 1. Therefore, the action-advice pair under the perfect information policy is given by

$$(\sigma_1(\theta), g(\theta)) = \begin{cases} (1,0) & \text{if } \theta = 1, \\ (0,0) & \text{if } \theta = 0, \end{cases}$$

and the principal's ex ante expected equilibrium payoff equals

$$\pi_1(\sigma \mid r, g) = p\{u_1(1,0) - c_1\} \equiv \pi_1^0. \tag{13}$$

Intuitively, if the agent knows that the principal knows the state, he will not choose $a_2 = 1$ because he knows that the principal chooses $a_1 = 1$ in state 1. Note that the above profile is equivalent to what happens when the principal acquires perfect information $\theta \in \{0, 1\}$ and then fully discloses it to the agent.

Comparison of the principal's payoff under no information in (12) and that under perfect information in (13) is summarized in the following proposition and is illustrated in Figure 1.

Proposition 1 *1. If $p < \frac{c_2}{d_2^0}$ or $p > \frac{c_1}{d_1^0}$, then perfect information yields the higher expected payoff to the principal than no information.*

2. If $p \in \left(\frac{c_2}{d_2^0}, \frac{c_1}{d_1^0}\right)$, then no information yields the higher expected payoff than perfect information.

Intuitively, no information dominates perfect information if the prior p is in the intermediate range so that under no information, the principal can commit to no action and induce the agent to take action unilaterally. When p is above this range, the shock is too likely for the principal to commit to no action, and when it is below this range, the shock is too unlikely for the agent to take action even unilaterally.

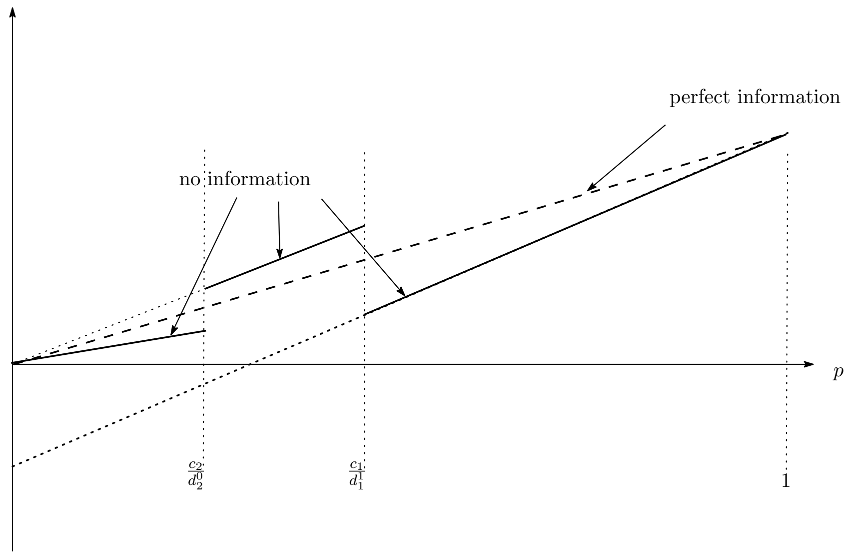

Figure 1: Principal's payoffs under perfect information and no information

In these cases, perfect information is better than no information. We can interpret the second observation in Proposition 1 as one expression of the value of strategic ignorance mentioned in Section 2. Our focus in subsequent sections is hence on the case where p is small so that complete ignorance is inferior to perfect information.

6 Obscurity with Finite Signals

In this section, we illustrate the benefit of imperfect information in simple models with finite signals. These examples show that the principal chooses his forecasting technology so as to control the likelihood ratio of the two states given each signal realization. In particular, he is interested in creating a signal realization that works as a commitment device to implement $(a_1, a_2) = (0, 1)$.

Given the conclusion of the previous section, suppose that the prior probability p of the shock state $\omega = 1$ is low and satisfies

$$p < \frac{c_2}{d_2^0} \quad \Leftrightarrow \quad \frac{p\delta_2^0}{1-p} < 1. \tag{14}$$

This in particular implies that the agent does not take action even unilaterally under no information.

Suppose first that the principal observes a binary signal $\theta \in \Theta = \{\ell, h\}$ $(\ell < h)$. For $r \in \left(0, \frac{1}{2}\right)$, the distribution of θ conditional on ω is given by

	$\omega = 1$	$\omega = 0$
$\theta = h$	$1 - r$	r
$\theta = \ell$	r	$1 - r$

The monotone likelihood ratio property (5) holds since $r < \frac{1}{2}$. As

$$\frac{f_{0,r}(\ell)}{f_{1,r}(\ell)} = \frac{1-r}{r} > 1 > \frac{p\delta_2^0}{1-p} > \frac{p\delta_1^0}{1-p},$$

the principal never chooses $a_1 = 1$ at $\theta = \ell$: $\sigma_1(\ell) = 0$. We hence consider the following two cases depending on whether he takes action at $\theta = h$ or not.

1) $\frac{f_{0,r}(h)}{f_{1,r}(h)} = \frac{r}{1-r} < \frac{p\delta_1^1}{1-p}$.

In this case, $a_1 = 1$ is dominant for the principal at $\theta = h$ by (9), but he cannot induce action $a_2 = 1$ from the agent: If the agent learns that $\theta = \ell$, he will choose $a_2 = 0$ since

$$\frac{f_{0,r}(\ell)}{f_{1,r}(\ell)} > \frac{p\delta_2^0}{1-p},$$

and if he learns that $\theta = h$, he will again choose $a_2 = 0$ since

$$\frac{f_{0,r}(h)}{f_{1,r}(h)} > 0 > \frac{p\delta_2^1}{1-p}.$$

Therefore, whether θ is (partially) revealed or not, the agent will choose $a_2 = 0$, and the principal cannot do better than choosing perfect information $r = 0$.

2) $\frac{p\delta_1^1}{1-p} \leq \frac{f_{0,r}(h)}{f_{1,r}(h)} = \frac{r}{1-r} \leq \frac{p\delta_2^0}{1-p}$.

In this case, the principal can commit to no action $a_1 = 0$ and induce the agent to choose $a_2 = 1$ at $\theta = h$. Suppose then that the action-advice pair is given by

$$(\sigma_1(\theta), g(\theta)) = \begin{cases} (0,1) & \text{if } \theta = h, \\ (0,0) & \text{if } \theta = \ell. \end{cases}$$

(r, g) is incentive compatible for the given range of r, and yields

$$\pi_1(r, g) = p\left[u_1(0,1)(1-r) + u_1(0,0)r\right].$$

Since $\pi_1(r,g)$ is decreasing in r, it is maximized when $\frac{r}{1-r}$ is at the lower end of the interval:

$$\frac{r}{1-r} = \frac{p\delta_1^1}{1-p} \quad \Leftrightarrow \quad r = \frac{\frac{p\delta_1^1}{1-p}}{1+\frac{p\delta_1^1}{1-p}}. \tag{15}$$

It follows that (r,g) dominates perfect information if

$$u_1(0,1)(1-r) + u_1(0,0)r - u_1(1,0) + c_1 > 0 \quad \Leftrightarrow \quad \frac{p\delta_1^1}{1-p} < \frac{\mu_1^0}{\delta_1^0} - 1.$$

When this holds, the optimal technology r in (15) is approximately proportional to the prior probability p when it is small.

In this problem, the principal chooses the technology r to directly control the likelihood ratio $\frac{f_{0,r}(h)}{f_{1,r}(h)}$ at signal h so that he would be just indifferent between taking action or not at $\theta = h$, and that $a_2 = 1$ would be a best response for the agent against $a_1 = 0$.[13] This way, the principal uses $\theta = h$ as a commitment device and induces the agent to take action. In reality, however, no forecasting technology is likely to place a bound on the likelihood ratio. In other words, under any kind of technology, there typically exists a signal realization that indicates the occurrence of a shock very strongly. Consider next a model with three signals ℓ, m and h ($\ell < m < h$) that captures such a possibility. Let a constant $\kappa \in (0,1)$ be given, and suppose that the technology r consists of two variables x and y with $0 \le x \le 1$ and $0 \le y \le \kappa$. The distribution of θ conditional on ω is given by

	$\omega = 1$	$\omega = 0$
$\theta = h$	$1-x$	0
$\theta = m$	κx	y
$\theta = \ell$	$(1-\kappa)x$	$1-y$

The technology parameters x and y both control the accuracy of the signals m and ℓ. On the other hand, $\theta = h$ perfectly indicates $\omega = 1$ regardless of x and y. Note that $x = 0$ corresponds to perfect information since $\omega = 0$ results in $\theta = \ell$ or m,

[13] The optimal policy when we allow random disclosure would advise $a_2 = 1$ with positive probability at $\theta = \ell$ so that the agent's incentive conditions would hold with equality. We do not consider such policies partly because they lack credibility. In the continuous signal model considered in Sections 7 and 8, this omission places no restriction since no random policies are optimal: for any random policy, there exists a superior deterministic policy that advise $a_2 = 1$ with the same ex ante probability but for higher signal realizations.

and $\omega = 1$ results in $\theta = h$. The monotone likelihood ratio property (5) holds when $y \leq \kappa$. Since

$$\frac{f_{0,r}(h)}{f_{1,r}(h)} = 0 < \frac{p\delta_1^1}{1-p} \quad \text{and} \quad \frac{f_{0,r}(\ell)}{f_{1,r}(\ell)} = \frac{1-y}{(1-\kappa)x} \geq 1 > \frac{p\delta_1^0}{1-p},$$

the principal chooses $\sigma_1(h) = 1$ and $\sigma_1(\ell) = 0$ as his dominant actions. If $\sigma_1(m) = 1$, the agent has no incentive to choose $a_2 = 1$ for each signal realization since

$$\frac{f_{0,r}(h)}{f_{1,r}(h)}, \frac{f_{0,r}(m)}{f_{1,r}(m)} \geq 0 > \frac{p\delta_2^1}{1-p} \quad \text{and} \quad \frac{f_{0,r}(\ell)}{f_{1,r}(\ell)} > \frac{p\delta_2^0}{1-p}.$$

It follows that if $\sigma_1(m) = 1$, no disclosure rule can induce the agent to choose $a_2 = 1$. Hence, let $\sigma_1(m) = 0$, and suppose that the action-advice pair is given by

$$(\sigma_1(\theta), g(\theta)) = \begin{cases} (1,0) & \text{if } \theta = h, \\ (0,1) & \text{if } \theta = m, \\ (0,0) & \text{if } \theta = \ell. \end{cases}$$

Since $g(\theta) = 1$ reveals that $\theta = m$, (r, g) is incentive compatible if

$$\frac{f_{0,r}(m)}{f_{1,r}(m)} = \frac{y}{\kappa x} \in \left[\frac{p\delta_1^1}{1-p}, \frac{p\delta_2^0}{1-p}\right],$$

and the principal's expected payoff under (r, g) is given by

$$\pi_1(r, g) = \pi_1^0 + pc_1 x \left(\kappa\mu_1^0 - \delta_1^0\right).$$

Therefore, it dominates perfect information when $\kappa > \frac{\delta_1^0}{\mu_1^0}$.[14] In this scheme, the principal uses the medium risk signal $\theta = m$ as a commitment device. Furthermore, since his payoff is increasing in x, the principal would set x as high as possible: He attempts to minimize the probability of observing the perfect signal $\theta = h$. Note that the argument does not rely on the pooling of observed information since the strategy profile (σ_1, σ_2^*) is implementable when θ is fully disclosed.

7 Obscurity with Continuous Signals

With a continuous signal, we can describe a more realistic environment in which the likelihood ratio is unbounded, but is finite and strictly positive so that no signal

[14] The optimal policy with random disclosure would advise $a_2 = 1$ with positive probability when $\theta = \ell$ and/or $\theta = h$.

perfectly reveals the state. In this section, we suppose that the signal θ is given by

$$\theta = \omega + r\epsilon,$$

where ϵ is a random noise term independent of ω. For concreteness, we suppose that ϵ has the standard normal distribution $N(0,1)$ whose cumulative distribution and density are denoted by Φ and ϕ, respectively. It follows that when the principal adopts technology $r > 0$, his signal θ has the normal distribution with mean ω and variance r^2 in state ω. We will identify a sufficient condition for the existence of an imperfect information policy (r, g) that dominates perfect information.

Theorem 2 *Suppose that*

$$\mu_1^0 - \delta_1^0 - \delta_1^1 > \left(\frac{\delta_1^1}{\delta_2^0}\right)^{\frac{1}{2}} \mu_1^0. \tag{16}$$

Then there exists $r > 0$ such that for the action advice pair (σ_1, g) specified below, the incomplete information policy (r, g) is incentive compatible with an equilibrium (σ_1, σ_2^*), and dominates perfect information.

$$(\sigma_1(\theta), g(\theta)) = \begin{cases} (1,0) & \text{if } \theta \geq \beta, \\ (0,1) & \text{if } \theta \in [\alpha^0, \beta), \\ (0,0) & \text{if } \theta < \alpha^0, \end{cases}$$

where β and α^0 are as defined in (7) and (8), respectively.

The information policy in the theorem is illustrated in Figure 2.

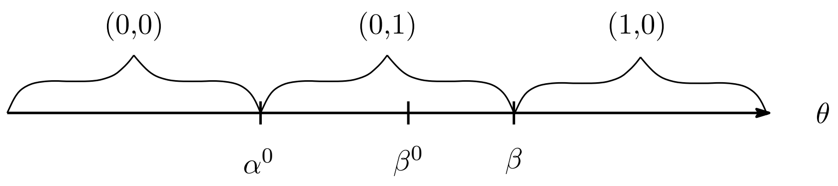

Figure 2: Action-advice pairs in Theorem 2

This policy is equivalent to the full disclosure of θ since it advises $a_2 = 1$ if and only if $\theta \in (\alpha^0, \beta)$. Hence, pooling of information is not essential for the obscurity result and even when the principal is constrained to full disclosure, he should optimally acquire imperfect information.

Corollary 3 *Under the conditions of the theorem, there exists $r > 0$ such that full disclosure of $\theta = \omega + r\epsilon$ yields a strictly higher payoff to the principal than perfect information.*

The intuition behind the theorem and its corollary is as follows: As is the case with the three-signal model of the previous section, the principal in the continuous signal model uses the medium risk signal as a commitment device to implement $(a_1, a_2) = (0, 1)$. Unlike before, however, the exact range of the medium risk signals is endogenously determined by the technology parameter r. The cost of imperfect information for the principal is the cost of wasted effort in state 0. On the other hand, the benefit is the existence of such medium risk signals. The condition (16) ensures that the probability that the signal falls in this medium risk range is sufficiently large compared with the probability of observing a high signal in state 0. Note that (16) is strictly in terms of the payoffs and independent of the prior probability p. It tends to hold (1) when δ_2^0 is large so that the agent has a strong incentive to make effort in the absence of the principal's effort, or (2) when the agent's action has a significant positive impact on the principal's payoff. For the latter, suppose for example that the principal's payoff is written in the form:

$$u_1(a_1, a_2) = k_1 a_1 + k_2 a_2 - a_1 a_2,$$

where the cross product term represents strategic substitution. Then the benefit of the agent's action for the principal equals $\mu_1^0 = \frac{u_1(0,1) - u_1(0,0)}{c_1} = \frac{k_2}{c_1}$, whereas that of his own action δ_1^0 or δ_1^1 does not depend on k_2. Hence, (16) tends to hold as we increase k_2 while holding other parameters fixed.

The above conclusion extends to alternative setups where the principal and the agent move sequentially: Suppose first that the agent moves first. In this case, the agent's inaction would force the principal to take action for the medium risk signals. However, if the agent is in fact a continuum of individuals none of whom can influence the principal's decision, then the above conclusion holds as is. Such interpretation of the agent is appropriate for the type of shock considered in this paper. Suppose next that the principal moves first. In this case, the principal's action may signal his private information in addition to what is revealed through information disclosure. This problem, however, is not relevant as long as full disclosure is considered.[15]

[15] The optimal disclosure rule considered in the next section requires some modification since $(a_1, a_2) = (1, 1)$ cannot be implemented for $\theta > \gamma$.

When full disclosure of θ is assumed as in the corollary above, we can see under (14) and (16) that the principal's payoff is single-peaked as a function of r^2, and hence that there exists a unique $r > 0$ that maximizes his payoff. Specifically, the optimal r is characterized as a solution to the following equation, which is the first-order condition with respect to r^2 for the maximization of the principal's payoff π_1 in (20) in the Appendix:

$$\left(\frac{\delta_1^1}{\delta_2^0}\right)^{\frac{1}{2}\left(1+r^2\log\left(\frac{p}{1-p}\right)^2\delta_2^0\delta_1^1\right)} = \frac{\mu_1^0-\delta_1^0-\delta_1^1-2r^2(\mu_1^0-\delta_1^0+\delta_1^1)\log\frac{p\delta_1}{1-p}}{\mu_1^0\left(1-2r^2\log\frac{p\delta_2^0}{1-p}\right)}. \tag{17}$$

The LHS is convex in r^2 while the RHS is concave in r^2. Moreover, the LHS > RHS at $r^2 = 0$ under (16). These imply that the principal's payoff π_1 is quasi-concave in r^2, and hence that there exists a unique $r^2 > 0$ which maximizes his payoff. It is interesting to compare this observation with the no-existence of an interior optimum in a single-person information acquisition problem as implied by the theory of the value of information.[16] Another interesting observation concerns the comparative statics with respect to p. First, $r \to 0$ as $p \to 0$ since otherwise, the principal's payoff under (r, g) in (20) will become smaller than π_1^0. Let

$$z = \frac{1}{2} - r^2\log\frac{p}{1-p},$$

which is $> \frac{1}{2}$ when $p < \frac{1}{2}$. The first-order condition (17) can be written in terms of r^2 and z as:

$$\begin{aligned}&\left(\mu_1^0-\delta_1^0\right)\left(z-r^2\log\delta_1^1\right)-\delta_1^1\left(1-z+r^2\log\delta_1^1\right)\\ &-\mu_1^0\left(z-r^2\log\delta_2^0\right)\left(\frac{\delta_1^1}{\delta_2^0}\right)^{1-z+\frac{1}{2}r^2\log\delta_1^1\delta_2^0}=0.\end{aligned}$$

Suppose that $p \to 0$ so that $r^2 \to 0$. Then in the limit, z should satisfy

$$\left(\mu_1^0-\delta_1^0\right)z-\delta_1^1(1-z)-\mu_1^0 z\left(\frac{\delta_1^1}{\delta_2^0}\right)^{1-z}=0,$$

which is seen to have a solution in the interval $\left(\frac{1}{2}, 1\right)$ under (16). In other words, when p is small, z is close to the solution to this equation. Put differently, for p small,

$$r^2\log\frac{p}{1-p}$$

[16]The theory states that the value of information is fundamentally non-concave. See Radner and Stiglitz (1984).

is approximately constant, and hence

$$r \propto \left(\log \frac{p}{1-p}\right)^{-\frac{1}{2}}.$$

It also follows from this that the thresholds α^0 and β are held approximately constant as $p \to 0$.

8 Optimal Disclosure with Imperfect Continuous Signals

Given the conclusion of the previous sections that an imperfect information technology may be optimally chosen, we characterize an optimal disclosure rule when the imperfect signal θ of given precision is continuously distributed over the real line $\Theta = \boldsymbol{R}$. In other words, we examine how the principal should optimally pool his private information. By solving for the optimal disclosure rule for each $r > 0$ using Theorem 4 below, we can in principle solve for the optimal technology r^* at least numerically. The theorem is of interest in its own right when acquiring perfect information is technologically infeasible.

Assume that the likelihood ratio strictly decreases from ∞ to 0 as θ varies from 0 to 1 so that α^0, β^0 and β defined in (7) and (8) uniquely exist. The following theorem characterizes an optimal disclosure rule g and the associated equilibrium σ given $r > 0$.

Theorem 4 *Suppose that (r, g) is an optimal incentive compatible policy with imperfect information, and admits an equilibrium $\sigma = (\sigma_1, \sigma_2^*)$. Let α and γ be the solutions to the following minimization problem:*

$$\begin{aligned} &\min_{\alpha,\gamma} \mu_1^0 F_{1,r}(\alpha) + \mu_1^1 F_{1,r}(\gamma) \\ &\text{subject to : } \alpha \leq \alpha^0,\ \gamma \geq \beta \\ &\qquad \frac{p}{1-p}\left[P(\theta \in [\alpha,\beta) \mid \omega = 1)\delta_2^0 + P(\theta \in [\gamma,\infty) \mid \omega = 1)\delta_2^1\right] \\ &\qquad\qquad = P(\theta \in [\alpha,\beta) \cup [\gamma,\infty) \mid \omega = 0). \end{aligned} \tag{18}$$

Then the pair (σ_1, g) *is given by*

$$(\sigma_1(\theta), g(\theta)) = \begin{cases} (0,0) & \textit{for almost every } \theta \in (-\infty, \alpha), \\ (0,1) & \textit{for almost every } \theta \in [\alpha, \beta), \\ (1,0) & \textit{for almost every } \theta \in [\beta, \gamma), \\ (1,1) & \textit{for almost every } \theta \in [\gamma, \infty). \end{cases}$$

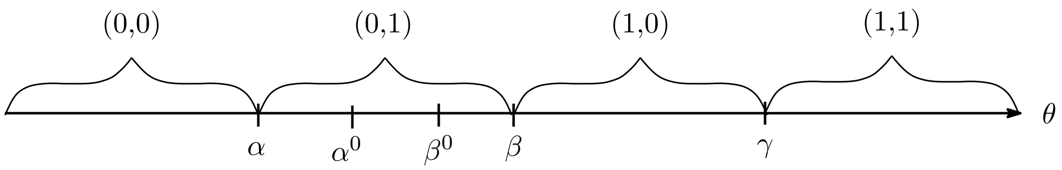

Figure 3: Action-advice pairs under the optimal imperfect information policy

Proposition 4 is illustrated in Figure 3. Although its formal proof in the Appendix is lengthy, the intuition behind the proposition is very simple. To see this, suppose that the principal fully reveals θ. When $\alpha^0 < \theta < \beta$, the agent is willing to take action as a best response to no action by the principal. In other words, the principal can induce $a_2 = 1$ *for free* over this range. When $\theta < \alpha^0$ or when $\theta > \beta$, on the other hand, the agent does not take action since in the first case, the implied probability of $\omega = 1$ is too low and in the second case, he knows that the principal takes action. Hence, with full disclosure, the agent can be induced to take action if and only if $\theta \in (\alpha^0, \beta)$. In order to have the agent take action even when $\theta < \alpha^0$ or $\theta > \beta$, the principal must garble information appropriately. The extent of garbling is determined by the agent's incentive condition (10), and at the optimum, the constraint should bind as expressed by the equality constraint in the (18). Furthermore, in each of the two intervals $(-\infty, \alpha^0)$ and $[\beta, \infty)$, the agent is advised to take action when the signal is in its upper portion. Intuitively, this is because the higher the signal θ, the more effective is the agent's action in increasing the principal's payoff. Finally, given such specification of the action-advice pair

(σ_1, g), the principal's payoff can be written as

$$\begin{aligned}
\pi_1(r,g) &= p\Big[u_1(1,1)\{1-F_{1,r}(\gamma)\} + u_1(1,0)\{F_{1,r}(\gamma) - F_{1,r}(\beta)\} \\
&\quad + u_1(0,1)\{F_{1,r}(\beta) - F_{1,r}(\alpha)\} + u_1(0,0)F_{1,r}(\alpha)\Big] \\
&\quad - c_1\Big[p\{1-F_{1,r}(\beta)\} + (1-p)\{1-F_{0,r}(\beta)\}\Big] \\
&= p\{u_1(1,1) - c_1\} - c_1(1-p)\{1-F_{0,r}(\beta)\} \\
&\quad - pc_1\left[\mu_1^1 F_{1,r}(\gamma) + (\delta_1^0 - \mu_1^0)F_{1,r}(\beta) + \mu_1^0 F_{1,r}(\alpha)\right].
\end{aligned} \tag{19}$$

Since β is uncontrolable, π_1 is maximized when we minimize the objective function in (18) with respect to α and γ subject to the inequality constraints.

9 Conclusion

In a model of information acquisition and disclosure, we show that endogenous information about the risk of a shock may be imperfect when the agent may free ride on the principal's preparation efforts. For a shock with moderately high prior probability, the principal prefers no information to perfect information. On the other hand, for a shock with small prior probability, the principal prefers perfect information to no information, but the optimal policy may entail a strictly positive degree of imperfection. Specifically, we show that the full disclosure of imperfect information may outperform perfect information.

The model adopts an extreme assumption that a perfectly informative signal is costlessly available to the principal. Of course, if acquisition of more accurate information is more costly, then it only reinforces the main conclusion of the paper. When acquisition of perfect information is technologically infeasible, the relevant question is whether the optimal information is less precise than what is technologically feasible. The answer naturally depends on the parameters, but the basic intuition of the present analysis continues to be valid.

The scientific assessment of a risk is often very difficult to communicate to non-experts. Furthermore, it is often observed that individuals overreact to a small probability risk in some cases, and undermine a moderately high probability risk in other cases. In this sense, the biggest challenge for the sender of information may be to induce the right action from the receivers taking into account the imperfection and bias in their information processing.[17] Theoretical investigation into such a

[17] See, for example, Eggers and Fischhoff (2004) and Fischhoff (1994, 2011) for the discussion of

process would be an interesting topic of future research.

Appendix

Proof of Theorem 2 It is clear that (r, g) is incentive compatible since it advises $a_2 = 1$ if and only if $\theta \in (\alpha^0, \beta)$ where the principal chooses $a_1 = 0$. The principal's expected payoff under (r, g) is given by

$$\begin{aligned} &\pi_1(r,g) \\ &= p\Big[u_1(1,0)\,\{1 - F_{1,r}(\beta)\} + u_1(0,1)\,\{F_{1,r}(\beta) - F_{1,r}(\alpha^0)\} \\ &+ u_1(0,0)\,F_{1,r}(\alpha^0)\Big] - c_1\Big[p\{1 - F_{1,r}(\beta)\} + (1-p)\{1 - F_{0,r}(\beta)\}\Big]. \end{aligned} \tag{20}$$

Recalling that $\pi_1^0 = p\{u_1(1,0) - c_1\}$ is the principal's payoff under the perfect information policy, we hence have

$$\pi_1(r,g) - \pi_1^0 = c_1 p\left\{(\mu_1^0 - \delta_1^0)F_{1,r}(\beta) - \mu_1^0 F_{1,r}(\alpha^0)\right\} - c_1(1-p)\left\{1 - F_{0,r}(\beta)\right\}. \tag{21}$$

We now show that under (16),

$$\lim_{r\to 0} \frac{\pi_1(r,g) - \pi_1^0}{c_1 F_{1,r}(\beta)} > 0.$$

This would imply that $\pi_1(r,g) > \pi_1^0$ for $r > 0$ sufficiently small, showing the sub-optimality of the perfect information policy. Given that the noise term ϵ has the standard normal distribution, α^0 and β in (7) and (8) are explicitly given by

$$\alpha^0 = \frac{1}{2} - r^2 \log \frac{p\delta_2^0}{1-p}, \quad \beta = \frac{1}{2} - r^2 \log \frac{p\delta_1^1}{1-p},$$

where log denotes the natural logarithm. It follows that as $r \to 0$,

$$\frac{f_{1,r}(\alpha^0)}{f_{1,r}(\beta)} = e^{\left(-\frac{1}{2}\log\frac{\delta_2^0}{\delta_1^1}\right)\left\{1 + r^2 \log\left(\frac{p}{1-p}\right)^2 \delta_1^1 \delta_2^0\right\}} \to \left(\frac{\delta_1^1}{\delta_2^0}\right)^{\frac{1}{2}}. \tag{22}$$

We now show that

$$\lim_{r\to 0} \frac{1 - F_{0,r}(\beta)}{F_{1,r}(\beta)} = \frac{p\delta_1^1}{1-p}. \tag{23}$$

Since

$$\lim_{r\to 0} r^2 \left(\frac{\beta}{r}\right)' = -\frac{1}{2} \quad \text{and} \quad \lim_{r\to 0} r^2 \left(\frac{\beta - 1}{r}\right)' = \frac{1}{2},$$

communication strategies when the receivers have limited capabilities.

the definition of β in (7) implies that

$$\lim_{r\to 0}\frac{1-F_{0,r}(\beta)}{F_{1,r}(\beta)}=\lim_{r\to 0}\frac{1-\Phi(\frac{\beta}{r})}{\Phi(\frac{\beta-1}{r})}=\lim_{r\to 0}\frac{-rf_{0,r}(\beta)\left(\frac{\beta}{r}\right)'}{rf_{1,r}(\beta)\left(\frac{\beta-1}{r}\right)'}$$
$$=\frac{p\delta_1^1}{1-p}\lim_{r\to 0}\frac{-r^2\left(\frac{\beta}{r}\right)'}{r^2\left(\frac{\beta-1}{r}\right)'}=\frac{p\delta_1^1}{1-p},$$

where the second equality uses L'Hospital's rule as well as the fact that $\phi(\frac{\beta}{r})=rf_{0,r}(\beta)$ and $\phi(\frac{\beta-1}{r})=rf_{1,r}(\beta)$. Since

$$\frac{F_{1,r}(\alpha^0)}{F_{1,r}(\beta)}=\frac{\int_{-\infty}^{\alpha}f_{1,r}(\theta)\,d\theta}{\int_{-\infty}^{\beta}f_{1,r}(\theta)\,d\theta}<\frac{f_{1,r}(\alpha^0)}{f_{1,r}(\beta)},$$

it follows from (22) and (23) that as $r\to 0$,

$$\begin{aligned}\frac{\pi_1(r,g)-\pi_1^0}{c_1F_{1,r}(\beta)}&=p\left\{(\mu_1^0-\delta_1^0)-\mu_1^0\frac{F_{1,r}(\alpha^0)}{F_{1,r}(\beta)}\right\}-(1-p)\frac{1-F_{0,r}(\beta)}{F_{1,r}(\beta)}\\&>p\left\{(\mu_1^0-\delta_1^0)-\mu_1^0\frac{f_{1,r}(\alpha^0)}{f_{1,r}(\beta)}\right\}-(1-p)\frac{1-F_{0,r}(\beta)}{F_{1,r}(\beta)}\\&\to p\left\{(\mu_1^0-\delta_1^0)-\mu_1^0\left(\frac{\delta_1^1}{\delta_2^0}\right)^{\frac{1}{2}}-\delta_1^1\right\}.\end{aligned}$$

The limit is >0 if and only if (16) holds. ■

Proof of Theorem 4 Recall that the principal has a dominant action $a_1=1$ if $\theta>\beta$, and $a_1=0$ if $\theta<\beta^0$. Recall also that the best response is $a_1=1-a_2$ if $\theta\in(\beta^0,\beta)$. It follows that the action-advice pair $(\sigma_1(\theta),g(\theta))$ should satisfy

$$(\sigma_1(\theta),g(\theta))=\begin{cases}(1,0)\text{ or }(1,1) & \text{if }\theta>\beta,\\(1,0)\text{ or }(0,1) & \text{if }\theta\in(\beta^0,\beta),\\(0,0)\text{ or }(0,1) & \text{if }\theta<,\beta^0,\end{cases}\tag{24}$$

We first show that $(\sigma_1(\theta),g(\theta))=(0,1)$ for almost every $\theta\in(\alpha^0,\beta)$. Suppose that (r,g) is an incentive compatible policy such that $D_0=B_0\cap(\alpha^0,\beta)$ has positive measure. Consider an alternative policy $(r,\hat{g})$ such that

$$\hat{g}(\theta)=\begin{cases}1 & \text{if }\theta\in(\alpha^0,\beta),\\g(\theta) & \text{otherwise.}\end{cases}$$

Let also $\hat{\sigma}_1$ be given by

$$\hat{\sigma}_1(\theta) = \begin{cases} 0 & \text{if } \theta \in (\alpha^0, \beta), \\ \sigma_1(\theta) & \text{otherwise.} \end{cases}$$

We then have $\hat{B}_1 \equiv \{\theta : \hat{g}(\theta) = 1\} = B_1 \cup D_0$ and $\hat{B}_0 \equiv \{\theta : \hat{g}(\theta) = 0\} = B_0 \setminus D_0$. Since $\theta \in D_0$ implies $\theta > \alpha^0$, we have

$$\frac{f_{0,r}(\theta)}{f_{1,r}(\theta)} < \frac{p\delta_2^0}{1-p},$$

and hence

$$P(\theta \in D_0 \mid \omega = 1) = \int_{D_0} f_{1,r}(\theta) d\theta > \frac{1-p}{p\delta_2^0} \int_{D_0} f_{0,r}(\theta) d\theta = \frac{1-p}{p\delta_2^0} P(\theta \in D_0 \mid \omega = 0).$$

Using the incentive compatibility condition (10) for (r, g), we see that

$$\begin{aligned} &\delta_2^0 P(\theta \in \hat{B}_1 \cap (-\infty, \beta) \mid \omega = 1) + \delta_2^1 P(\theta \in \hat{B}_1 \cap (\beta, \infty) \mid \omega = 1) \\ &= \delta_2^0 P(\theta \in B_1 \cap (-\infty, \beta) \mid \omega = 1) + \delta_2^1 P(\theta \in B_1 \cap (\beta, \infty) \mid \omega = 1) \\ &+ \delta_2^0 P(\theta \in D_0 \mid \omega = 1) \\ &\geq \frac{1-p}{p} P(\theta \in B_1 \mid \omega = 0) + \frac{1-p}{p} P(\theta \in D_0 \mid \omega = 0) \\ &= \frac{1-p}{p} P(\theta \in \hat{B}_1 \mid \omega = 0). \end{aligned} \tag{25}$$

Likewise, the incentive compatibility condition (11) for (r, g) implies that

$$\begin{aligned} &\delta_2^0 P(\theta \in \hat{B}_0 \cap (-\infty, \beta) \mid \omega = 1) + \delta_2^1 P(\theta \in \hat{B}_0 \cap (\beta, \infty) \mid \omega = 1) \\ &= \delta_2^0 P(\theta \in B_0 \cap (-\infty, \beta) \mid \omega = 1) + \delta_2^1 P(\theta \in B_0 \cap (\beta, \infty) \mid \omega = 1) \\ &- \delta_2^0 P(\theta \in D_0 \mid \omega = 1) \\ &\leq \frac{1-p}{p} P(\theta \in B_0 \mid \omega = 0) - \frac{1-p}{p} P(\theta \in D_0 \mid \omega = 0) \\ &= \frac{1-p}{p} P(\theta \in \hat{B}_0 \mid \omega = 0). \end{aligned} \tag{26}$$

(25) and (26) show that $(r, \hat{g})$ is incentive compatible. The principal's payoff under $(r, \hat{g})$ differs from that under (r, g) only when $\theta \in D_0$: $(r, \hat{g})$ yields $pu_1(0, 1)$ while (r, g) yields $pu_1(1, 0) - c_1$ if $\theta \in D_0 \cap (\beta^0, \beta)$ and $pu_1(0, 0)$ if $\theta \in D_0 \cap (\alpha, \beta^0)$. Since $u_1(0, 1) > u_1(1, 0) - c_1 > u_1(0, 0)$ by assumption, $(r, \hat{g})$ yields a strictly higher payoff.

Formally,

$$\begin{aligned}\pi_1(r,\hat{g})-\pi_1(r,g) = p\Big[&u_1(0,1)P(\theta\in D_0\mid\omega=1)\\ &-u_1(0,0)P(\theta\in D_0\cap[\alpha,\beta^0)\mid\omega=1)\\ &-u_1(1,0)P(\theta\in D_0\cap[\beta^0,\beta)\mid\omega=1)\Big]\\ &+c_1P\left(\theta\in D_0\cap(\beta^0,\beta)\right)\\ &>0.\end{aligned}$$

This shows that (r,g) is suboptimal.

We next show that there exists $\gamma\in[\beta,\infty)$ such that $g(\theta)=0$ for almost every $\theta\in(\beta,\gamma)$ and $g(\theta)=1$ for almost every $\theta\in(\gamma,\infty)$. The proof of the existence of $\alpha\in(-\infty,\alpha^0]$ such that $g(\theta)=0$ for almost every $\theta\in(-\infty,\alpha)$ and $g(\theta)=1$ for almost every $\theta\in(\alpha,\alpha^0)$ is similar and is omitted.

We first claim that if (r,g) is optimal, then for any $x\in(\beta,\infty)$,

$$P(\theta\in(\beta,x)\cap B_1)>0\Rightarrow P(\theta\in(x,\infty)\cap B_0)=0. \tag{27}$$

Intuitively, (27) states that when g advises $a_2=1$ for some signal θ, then it should advise $a_2=1$ for almost every signal above θ. When (27) holds, there exists $\nu\in[\beta,\infty)$ such that $g(\theta)=1$ for almost every $\theta\in(\nu,\infty)$ and $g(\theta)=0$ for almost every $\theta\in(\beta,\nu)$ as follows: First, if $P((\beta,\infty)\cap B_1)=0$, then let $\nu=\beta$. Otherwise, if $P(\theta\in(\beta,x)\cap B_1)>0$ for some $x>\beta$, then let $\nu=\inf\{x:\ P(\theta\in(\beta,x)\cap B_1)>0\}$. By the definition of ν, we have $P(\theta\in(\beta,\nu)\cap B_1)=\lim_{n\to\infty}P(\theta\in(\beta,\nu-\frac{1}{n})\cap B_1)=0$. Furthermore, by (27), we have

$$P(\theta\in(\nu+\frac{1}{n},\infty)\cap B_0)=0 \text{ for every } n=1,2,\ldots$$

so that $P(\theta\in(\nu,\infty)\cap B_0)=\lim_{n\to\infty}P(\theta\in(\nu+\frac{1}{n},\infty)\cap B_0)=0$.

Suppose that (r,g) is an incentive compatible policy such that $P(\theta\in[\beta,x)\cap B_1)>0$ but $P(\theta\in(x,\infty)\cap B_0)>0$ for some $x\in(\beta,1)$. Take any $D_1^0\subset[\beta,x)\cap B_1$ and $D_0\subset(x,\infty)\cap B_0$ such that

$$P(\theta\in D_0\mid\omega=1)=P(\theta\in D_1^0\mid\omega=1)>0. \tag{28}$$

Since every element of D_0 is larger than any element of D_1^0, the monotone likelihood ratio property (5) implies that

$$\frac{P(\theta\in D_0\mid\omega=0)}{P(\theta\in D_0\mid\omega=1)}<\frac{P(\theta\in D_1\mid\omega=0)}{P(\theta\in D_1\mid\omega=1)}.$$

It then follows from (28) that

$$P(\theta \in D_0 \mid \omega = 0) < P(\theta \in D_1^0 \mid \omega = 0).$$

Hence,

$$\begin{aligned} 0 &= p\delta_2^1 \left\{ P(D_0 \mid \omega = 1) - P(D_1^0 \mid \omega = 1) \right\} \\ &> (1-p) \left\{ P(D_0 \mid \omega = 0) - P(D_1^0 \mid \omega = 0) \right\}. \end{aligned}$$

Now consider for any $\epsilon \in (-\infty, x-\beta)$ a subset D_1^ϵ of D_1^0 such that $D_1^\epsilon = D_1^0 \cap (\beta, x-\epsilon)$. Since both $P(\theta \in D_1^\epsilon \mid \omega = 1)$ and $P(\theta \in D_1^\epsilon \mid \omega = 0)$ are continuous functions of ϵ, for $\epsilon > 0$ small enough, we have

$$\begin{aligned} &p\delta_2^1 \left\{ P(\theta \in D_0 \mid \omega = 1) - P(\theta \in D_1^\epsilon \mid \omega = 1) \right\} \\ &> (1-p) \left\{ P(\theta \in D_0 \mid \omega = 0) - P(\theta \in D_1^\epsilon \mid \omega = 0) \right\}. \end{aligned} \tag{29}$$

Let $\epsilon > 0$ satisfy (29), and define $D_1 = D_1^\epsilon$. Note that we also have

$$P(\theta \in D_1 \mid \omega = 1) < P(\theta \in D_0 \mid \omega = 1). \tag{30}$$

Consider now an alternative policy $(r, \hat{g})$ such that

$$\hat{g}(\theta) = \begin{cases} 1 & \text{if } \theta \in D_0, \\ 0 & \text{if } \theta \in D_1, \\ g(\theta) & \text{otherwise,} \end{cases}$$

and let $\hat{\sigma}_1$ be given by

$$\hat{\sigma}_1(\theta) = \begin{cases} 0 & \text{if } \theta \in D_0, \\ 1 & \text{if } \theta \in D_1, \\ \sigma_1(\theta) & \text{otherwise.} \end{cases}$$

In other words, $\hat{g}$ and g advise the opposite actions when $\theta \in D_0 \cup D_1$. It follows that

$$\begin{aligned} \hat{B}_1 &\equiv \{\theta : \hat{g}(\theta) = 1\} = B_1 \cup D_0 \setminus D_1, \\ \hat{B}_0 &\equiv \{\theta : \hat{g}(\theta) = 0\} = B_0 \cup D_1 \setminus D_0. \end{aligned}$$

To see that $(r, \hat{g})$ is incentive compatible, note that the incentive compatibility condition (10) for (r,g) and (29) together imply that

$$\begin{aligned}
&\delta_2^0 P(\theta \in \hat{B}_1 \cap (-\infty,\beta) \mid \omega=1) + \delta_2^1 P(\theta \in \hat{B}_1 \cap (\beta,\infty) \mid \omega=1) \\
&= \delta_2^0 P(\theta \in B_1 \cap (-\infty,\beta) \mid \omega=1) + \delta_2^1 P(\theta \in B_1 \cap (\beta,\infty) \mid \omega=1) \\
&+ \delta_2^1 \{P(\theta \in D_0 \mid \omega=1) - P(\theta \in D_1 \mid \omega=1)\} \\
&> \frac{1-p}{p} P(\theta \in B_1 \mid \omega=0) + \frac{1-p}{p} \{P(\theta \in D_0 \mid \omega=0) - P(D_1 \mid \omega=0)\} \\
&= \frac{1-p}{p} P(\theta \in \hat{B}_1 \mid \omega=0).
\end{aligned} \tag{31}$$

Likewise, it follows from (11) and (29) that

$$\begin{aligned}
&\delta_2^0 P(\theta \in \hat{B}_0 \cap (-\infty,\beta) \mid \omega=1) + \delta_2^1 P(\theta \in \hat{B}_0 \cap (\beta,\infty) \mid \omega=1) \\
&= \delta_2^0 P(\theta \in B_0 \cap (-\infty,\beta) \mid \omega=1) + \delta_2^1 P(\theta \in B_0 \cap (\beta,\infty) \mid \omega=1) \\
&+ \delta_2^1 \{P(\theta \in D_1 \mid \omega=1) - P(\theta \in D_0 \mid \omega=1)\} \\
&< \frac{1-p}{p} P(\theta \in B_0 \mid \omega=0) + \frac{1-p}{p} \{P(\theta \in D_1 \mid \omega=0) - P(D_0 \mid \omega=0)\} \\
&= \frac{1-p}{p} P(\theta \in \hat{B}_0 \mid \omega=0).
\end{aligned} \tag{32}$$

(31) and (32) show that $(r,\hat{g})$ is incentive compatible.

The principal's payoff under $(r,\hat{g})$ differs from that under (r,g) only when $\theta \in D_0 \cup D_1$: (r,g) yields $pu_1(1,1)$ on D_1 and $pu_1(1,0)$ on D_0, whereas $(r,\hat{g})$ yields $pu_1(0,1)$ on D_1 and $pu_1(1,1)$ on D_0. Hence, by (30),

$$\begin{aligned}
&\pi_1(r,\hat{g}) - \pi_1(r,g) \\
&= p\Big[P(\theta \in D_0 \mid \omega=1)\{u_1(1,1) - u_1(1,0)\} \\
&+ P(\theta \in D_1 \mid \omega=1)\{u_1(1,0) - u_1(1,1)\}\Big] \\
&= p\{P(\theta \in D_0 \mid \omega=1) - P(\theta \in D_1 \mid \omega=1)\}\{u_1(1,1) - u_1(1,0)\} \\
&> 0.
\end{aligned}$$

This shows that (r,g) is suboptimal. If the agent's incentive constraint holds with strict inequality, then the principal can lower either α or γ to increase his payoff as seen in (19). ■

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