Information Feedback in a Dynamic Tournament*

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First draft: May 9, 2003
This version: April 5, 2007

Abstract

This paper studies the problem of information revelation in a multi-stage tournament where the agents' effort in each stage gives rise to a stochastic performance signal privately observed by the principal. The principal controls the agents' effort incentive through the use of a *feedback policy,* which transforms his private information into a public announcement. The optimal feedback policy is one that maximizes the agents' expected effort. The paper identifies when the principal should use the no-feedback policy that reveals no information, or the full-feedback policy that reveals all his information.

Key words: tournament, mechanism, information revelation, Jensen's inequality.
Journal of Economic Literature Classification Numbers: C72, D82.

*This is a substantially revised version of an earlier paper under the same title (Aoyagi (2005)). I am grateful to Drew Fudenberg, Hideshi Itoh, Atsushi Kajii, and Juuso Valimaki for useful comments in the revision process.

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1 Introduction

As a prominent form of relative performance evaluation, tournaments have attracted considerable attention in economic theory. The main focus of the theory is on the size and allocation of rewards that would maximize the performance of the competing agents, and on the comparison of the relative incentive schemes against more general forms of contracts. Beginning with the seminal work of Lazear and Rosen (1981), a partial list of the literature on this subject includes Green and Stokey (1983), Nalebuff and Stiglitz (1983), Glazer and Hassin (1988), Gradstein and Konrad (1999), Moldovanu and Sela (2001), and others. In most models, a tournament is described as a static mechanism in which the agents' one-time effort decision determines their performance and hence the winner. In reality, however, many tournaments are more appropriately described as dynamic games: Agents make sequential effort decisions in multiple stages and the winner is determined by their overall performance. One important consideration when designing a tournament as a dynamic mechanism concerns the control of information during the course of play. In other words, the design of a dynamic tournament should include the strategic planning of what information to make available to the participants at what stage. The mode of such information revelation will have a significant impact on the participants' effort incentive. This point is well exemplified by a tournament for job promotion within a firm: First, such a tournament is dynamic in nature and spans multiple stages. Second, workers' performance is often measured by subjective criteria such as leadership, originality, ability to work in teams, *etc.* Such information is most appropriately described as private information of their superior or the firm's personnel department, and the latter communicates this information back to the workers as a way of providing motivation. Research on performance management well recognizes that inducement of the work incentive requires careful designing of information feedback.[1]

In this paper, we formulate a model of a dynamic tournament in which the principal receives private information about agents' performance, and then reveals as a feedback some or all of his information to the agents. The analysis is dual to that in the standard contest literature in that we fix prizes and focus exclusively on the effects of information. While strategic transmission of private information is a much studied subject in economic theory, no general understanding exists about how a designer's private information should be incorporated into his mechanism.

[1] See, for example, Williams (1998).

It is important to understand that information feedback has two separate effects on the agents' incentives. First, the revealed information influences the agents' incentives by changing their beliefs. This is true irrespective of whether the principal's private information is given exogenously as in the case of the linkage principle, or is generated endogenously by the agents' own actions as in our model. We call this the *ex post* effect of information feedback. On the other hand, when the private information is generated endogenously, each agent will choose their actions strategically so as to influence the content of the revealed information. For example, agents may exert extra effort in early stages to take the leading position and discourage opponents. We call this the *strategic* effect of information feedback.

In our model of a multi-stage tournament, agents' performance in each stage is stochastically related to their effort in that stage. The principal *privately* observes their performance realization after each stage, and reveals some or all of his private information to the agents before the next stage. The principal's *feedback policy* transforms the raw observation of the agents' performance into a *public* announcement. In our terminology, the closed-loop and open-loop formats described above correspond to the *full-feedback* and *no-feedback* policies, respectively.[2] The principal is free to choose any feedback policy and publicly announces its use before the tournament. For example, he may declare the use of a hybrid policy that reveals full information for some signal realizations but no information for others.[3] We assume that the principal is committed to his feedback policy for any realization of the private signal. The optimal feedback policy is one that maximizes the principal's payoff which is an increasing function of the agents' expected efforts. As discussed below, we find that whether he should reveal more information or not depends critically on the functional form of the agents' disutility of effort.

A more detailed description of our model is as follows: Two agents compete in a tournament over two stages. The agent with the higher performance at the end of the second stage wins and is awarded a prize of a fixed value such as a promotion to a higher job rank. In each of the two stages, each agent chooses an effort level, which is observed by neither the principal nor his opponent. The agents' cost function of effort is *time-separable* and can be expressed as the sum of stage-cost functions, which are assumed to be all strictly convex. The score in each stage is the difference between the performance levels of the two agents and

[2]Alternatively, the no-feedback policy can be interpreted as the simultaneous implementation of multiple one-shot tournaments.

[3]Under such a policy, of course, "no announcement" also has an informational content.

equals the sum of the difference between their effort levels *and* a random noise term. The principal privately observes the score, and makes a public announcement about it at the end of stage 1. Conditional on the announcement, the agents form their inference about the score and decide on effort levels in the second stage. We study how the choice of a feedback policy affects the agents' effort levels in a pure perfect Bayesian equilibrium (PBE) of this dynamic game.

The paper presents sufficient conditions for the existence of a PBE and characterizes effort levels on the equilibrium path. Specifically, a pure PBE is shown to exist if the noise component of the performance score is sufficiently large. In particular, the required level of noise can be taken independent of the feedback policy. We then identify the optimal feedback policy based on the agents' expected effort in equilibrium. In short, revealing more information is better for the principal when the marginal cost of effort is concave, and the converse is true when the marginal cost is convex. More specifically, the following observations are made: When the stage 2 marginal cost function of effort is convex, the no-feedback policy is optimal in the class of feedback policies that admit a symmetric PBE. On the other hand, the full-feedback policy is optimal in the same class when the marginal cost function is concave. When the two agents' efforts are sufficiently complementary to each other in the principal's payoff function, the no-feedback and full-feedback policies are also optimal within the wider class of feedback policies that induce a possibly asymmetric PBE.

The intuition for the above conclusions is as follows: As is standard, the agents' effort choice in each stage balances its marginal disutility with the expected marginal increment in the probability of winning. In stage 2, note that the marginal increment in the probability of winning depends in general on the stage 1 score x_1 as well as the stage 2 efforts of both agents. The key observation is that in any PBE (symmetric or not), the two agents choose a symmetric effort profile in stage 2 following any announcement. With the stage 2 efforts of the two agents canceling each other out, the marginal increment in the probability of winning depends only on the stage 1 score x_1 in equilibrium. It follows that the expected marginal increment equals the expected value of a function of x_1 alone, where the expectation is conditional on the information partition generated by the feedback policy. Suppose, for the sake of discussion, that the feedback policy distinguishes between only two events $x_1 \in A$ and $x_1 \in B = \mathbf{R} \setminus A$. If $M(x_1)$ denotes the marginal increment in the probability of winning at x_1 in equilibrium, then the expected marginal increment given A

equals $E[M(\tilde{x}_1) \mid A]$, and that given B equals $E[M(\tilde{x}_1) \mid B]$. Ex ante, the expected marginal increment is the sum of these two values weighted by the probabilities of A and B, or equivalently, the unconditional expectation $E[M(\tilde{x}_1)]$. Note, however, that this is precisely equal to the marginal increment in equilibrium under the no-feedback policy. Generalizing this argument to any information partition, we find that the ex ante expected marginal disutility of effort is the same under any policy. Since effort itself is the inverse of the marginal disutility, it can be seen through Jensen's inequality that the convexity or concavity of the marginal disutility function dictates the relative magnitude of expected effort in stage 2. In stage 1, on the other hand, it can be shown somewhat surprisingly that the agents' effort is the same under any feedback policy as long as the PBE is symmetric. This result can be seen as follows. First, in any symmetric PBE, we can verify that the marginal disutility in stage 1 equals the expected marginal disutility in stage 2.[4] It then follows from the constancy of expected marginal disutility in stage 2 that marginal disutility in stage 1 is the same in symmetric PBE under any policy. This shows that the stage 1 effort in symmetric PBE is also independent of the choice of a feedback policy.

Given these observations, one interesting question concerns the optimal feedback policy when the marginal cost of effort is neither convex nor concave on the relevant domain. We attempt to answer this question by examining the marginal cost function having a single reflection point at which its curvature changes from convex to concave. Our candidate feedback policy reveals full information when the absolute value of the score is less than some threshold, and reveals nothing (other than the fact that the threshold has been exceeded) otherwise. We show that such a feedback policy outperforms the full-feedback policy. A similar argument proves that no-feedback policy is dominated by the feedback policy that only reveals whether or not the score has exceeded some threshold.

In some applications, it is more appropriate to suppose that the agents inherently know their own performance. In a promotion tournament, for example, if the performance of each agent is measured objectively by the number of products they have sold, then each agent learns his own performance even if no information feedback is provided.[5] The only option for the principal in this case is whether to reveal additional information to each agent on the performance of the other agent.

[4] Although intuitive, intertemporal equality of marginal disutility cannot be assumed *a priori* because of the strategic effect mentioned above. For example, agents may be willing to incur higher marginal disutility in stage 1 in order to preempt the leading position.

[5] I thank the associate editor for prompting me to think about such a possibility.

We analyze this problem in an alternative framework in which an agent's individual performance equals the sum of his own effort and a random noise term. Unlike before, one major difficulty in this problem is the possible asymmetry of the stage 2 effort profile caused by the asymmetry of information between the agents. We avoid this difficulty by assuming that the performance noise in stage 1 can take only one of two values for each agent and that the stage 2 effort can influence the outcome only when the stage 1 noise is the same for both agents. In this setup, we analyze the expected effort induced under *private feedback* where the agents observe only their own performance, and then compare it with those under the full-feedback and no-feedback policies.[6] We show that when the stage 2 marginal cost function is convex, private feedback induces a higher expected effort from each agent than the full-feedback policy but lower effort than the no-feedback policy. When the marginal cost is concave, on the other hand, full-feedback induces the highest expected effort followed by private feedback and no-feedback in this order. The stage 1 effort is the same under the three policies. It is interesting to note that the expected effort is related to the amount of revealed information in the same monotone manner as in the original model.

While most of the analysis in this paper is on pure equilibria, it is interesting to examine what happens when agents mix their effort choices. Intuitively, we may consider a mixed choice of effort in stage 1 as an expression of the strategic effect discussed earlier. In other words, an agent in stage 1 may mix over different effort levels and choose a costly effort with positive probability because it can result in the possible reduction in stage 2 effort through the discouragement of his opponent. In a simplified environment with binary effort choices in stage 1, we show that this is exactly what happens in a mixed equilibrium under the full-feedback policy. This suggests that the stage 2 effort in a mixed PBE is on average lower than that in a pure PBE. When the strategic effect of information feedback takes the form of a mixed equilibrium, hence, the full-feedback policy may become less desirable in comparison with the no-feedback policy.

As discussed below, existing theories provide varying intuitions on the optimal degree of information revelation.

In auction theory, the so-called *linkage principle* by Milgrom and Weber (1982) asserts that under the affiliated distribution of signals, the seller's expected revenue

[6] The no-feedback policy represents the hypothetical scenario where the agents do not even observe their own performance.

is the highest when he is committed to revealing all of his private information to the bidders.[7] In a related framework, Milgrom (1981) shows that the seller of a good maximizes his payoff by revealing all his private information to the buyer if it is affiliated with the quality of his good. In some other situations, however, it is shown that the intuition furnished by the linkage principle fails to hold: Kaplan and Zamir (2000) analyze the problem of an auctioneer privately informed about bidders' valuations. In an independent private values framework, they find that the auctioneer is better off revealing the maximum of the valuations than fully revealing his information. In a model of twice-repeated common-value auctions with affiliated signals, de-Frutos and Rosenthal (1998) show that the auctioneer's expected revenue (over two auctions) is lower when information about stage 1 bids is made public than when it is not.[8]

The literature on dynamic models of a race also provides a closely related observation in the discussion of the closed- and open-loop formats.[9] The open-loop format reveals no information to the players during a competition, whereas the closed-loop format reveals the competitors' positions publicly and instantaneously. It is often argued that the players tend to slack off in the closed-loop format since, when one player has a lead over the others, the followers cannot catch up with the leader (in expected terms) by making the same level of effort as him. For example, Fudenberg *et al.* (1983) demonstrate the phenomenon of ε-preemption, where players stop making effort as soon as one of them establishes a small lead over others.

The problem of an agent's effort incentive and information is studied mainly in the context of dynamic principal-agent models. In the analysis of a repeated principal-agent game with a public performance signal, Radner (1985) considers a review strategy for the principal that evaluates the agent's performance at the end of each review phase that spans a large number of periods. He notes that inefficiency is inevitable as the agent relaxes near the end of the review phase if he realizes that his effort no longer influences the outcome of the review. Lizzeri *et al.* (2002) study a two-stage principal-agent problem where the agent's performance information is the principal's private information as in the present paper. Comparison is made on the agent's effort and the principal's payoff when the stage 1 performance (which

[7]A probability distribution is *affiliated* if the joint density function is log-supermodular.

[8]Perry and Reny (1999) report the failure of the linkage principle in a multi-object auction based on an entirely different logic.

[9]See, for example, Harris and Vickers (1985), and Fudenberg *et al.* (1983). Radner (1985) also makes a related observation in the context of a repeated principal-agent game.

is either a success or a failure) is revealed to the agent and when it is not.[10] They find that the revelation of information leads to a higher expected effort under the fixed wage profile and a quadratic cost function, but that the no-revelation scheme implements the same expected effort less expensively when the wage profile itself can be adjusted simultaneously. This independent finding by Lizzeri *et al.* (2002) complements the present analysis although no direct comparison is possible because of the differences in the modeling choice over the number of agents and variability of the reward.[11]

It is also instructive to interpret our conclusion in the light of the generalized revelation principle in mechanism design as formulated by Myerson (1982).[12] The principle asserts that when the principal wants to induce an optimal course of action from the agents as a function of their private information, he can restrict attention to a direct revelation-suggestion mechanism in which reporting of private information by the agents and the suggestion of actions by the principal are each done in a single step and hence involve no exchange of information among the agents. At a first glance, hence, this principle may seem to imply the optimality of no-feedback. It should be noted, however, that suggestion of actions to the agents, whether it is carried out publicly or privately, is itself a form of information feedback: the principal chooses a suggestion to each agent as a function of other agents' private information. The question of optimal information feedback in this context is hence how much functional relationship the principle should allow between the suggestion to any agent and the information solicited from others. The revelation principle provides no answer to this question. In terms of the modeling choice, of course, the major difference is that we assume free acquisition of private information by the principal, and instead focus on its strategic release.

As seen above, our theory of information feedback highlights the role of the third derivative of the agents' disutility of effort. It should be noted that economic theory often points out the relevance of the higher-order derivatives in incentive problems. In consumption theory, for example, it is well known that the precautionary saving motive in the face of future uncertainty is characterized by the third derivative of

[10] Given the binary nature of the private signal, these are the only (deterministic) feedback policies in Lizzeri *et al.* (2002).

[11] Although out of the scope of this paper, one natural question concerns the optimal scheme when the principal also controls the size and allocation of the reward in the present framework.

[12] I thank an anonymous referee for suggesting the possible connection between the revelation principle and information feedback.

an individual's utility function: Future uncertainty induces more current savings when the stage marginal utility is convex, and less savings when it is concave.[13] As another example, Rogerson (1985) shows in a two-stage principal-agent model with public information that a condition involving the third derivative of the agent's utility function determines whether the expected wage should rise or fall over time. In mechanism design, the optimality of a deterministic mechanism is also associated with the third-order cross derivatives of an agent's utility function.[14] The major difficulty in interpreting the results involving the higher-order derivatives is that there is perhaps no common consensus on what signs they assume. In this sense, the theory developed in this paper should be interpreted on the contingent basis as follows. First, if we think that the common specification of the quadratic cost function is a good approximation of the reality, then the theory predicts the insignificance of information feedback: With quadratic disutility, in fact, not only are the no-feedback and full-feedback policies optimal, but so is *any* feedback policy. When the third derivative is estimated to be close to zero, hence, the efficiency loss from using a suboptimal policy would be negligible if any. On the other hand, if we believe that the third derivative is significantly different from zero, the theory shows that information feedback is crucial. For example, one immediate implication of the theory is that an optimal policy under concave marginal disutility is least desirable under convex marginal disutility, and vice versa. In general, optimizing over feedback policies can lead to a substantial improvement in the induced expected effort.[15] No matter which assumption on the third derivative one thinks is more appropriate in each context, however, it should be emphasized that the evaluation of information feedback programs requires a proper theory. This paper provides a starting point for such analysis: It shows when information feedback has a crucial consequence, and that what may otherwise be considered an irrelevant detail can have a large impact in these problems.[16]

The paper is organized as follows: In the next section, we formulate a model of a dynamic tournament. Section 3 characterizes a PBE and provides sufficient conditions for its existence. Optimal feedback policies are studied in Section 4.

[13]See, for example, Kimball (1990, 1993).

[14]See, for example, Fudenberg and Tirole (1991).

[15]Reversing the argument, we may possibly use the theory to estimate the third-order derivatives.

[16]It is perhaps misleading to emphasize just the third derivatives. For example, in a related model with discrete effort levels and hence with no derivatives, we find that the optimality of a feedback policy depends rather subtly on the probability distribution of the noise term.

Section 5 analyzes private feedback and Section 6 studies mixed equilibria. We conclude in Section 7.

2 Model of a Tournament

Two risk neutral agents $i = 1, 2$ compete in two stages. In each stage, the agents' effort gives rise to a stochastic *performance score,* which indicates the difference in their performance. At the end of stage 2, the principal aggregates the scores from the two stages to determine the winner.

Formally, suppose that agent i's effort a_t^i in stage t is chosen from the set $\mathbf{R}_+$ of non-negative real numbers. The *stage t score* x_t is a random variable whose distribution depends on the effort levels a_t^1 and a_t^2 of both agents in stage t. More specifically, we assume that $x_t = a_t^1 - a_t^2 + \zeta_t$ for a real-valued random variable ζ_t. In other words, the score x_t represents agent 1's lead over agent 2, and is stochastically related to the difference between their effort levels. Let ϕ_t be the density of ζ_t over $\mathbf{R}$, and denote by Φ_t the corresponding cumulative distribution. We assume that ϕ_t is strictly positive and twice continuously differentiable, and symmetric around zero in the sense that $\phi_t(x) = \phi_t(-x)$ for any $x \in \mathbf{R}$. We also assume that ζ_1 and ζ_2 are independent. Note that the density of x_t under the action profile $a_t = (a_t^1, a_t^2)$ is given by

$$\phi_t(x_t - a_t^1 + a_t^2).$$

The (aggregate) *score* x is the sum of scores in stages 1 and 2: $x = x_1 + x_2$. Agent 1 wins if $x > 0$, and agent 2 wins if $x < 0$. Each agent wins with equal probability in the (probability zero) event of a tie $x = 0$. The sum of the two random variables ζ_1 and ζ_2 represents the noise in the aggregate score. If we denote its density by ϕ, then it can be expressed in terms of ϕ_1 and ϕ_2 as

$$\bar{\phi}(x) = \int_{\mathbf{R}} \phi_1(x-u)\,\phi_2(u)\,du. \tag{1}$$

This $\bar{\phi}$ is known as the *convolution* of ϕ_1 and ϕ_2, and is often denoted $\phi_1 * \phi_2$.

Each agent derives one unit of positive utility from winning the prize (*e.g.,* promotion to a higher job rank), and incurs disutility from effort. The cost of effort in stage t is described by a twice differentiable cost function $c_t : \mathbf{R}_+ \to \mathbf{R}_+$. Accordingly, agent i's overall utility equals $1 - \sum_{t=1}^2 c_t(a_t^i)$ if he wins, and $-\sum_{t=1}^2 c_t(a_t^i)$ otherwise. Throughout, we assume that in each stage $t = 1, 2$, the marginal cost of effort equals zero at no effort, and is strictly increasing:

$$c_t'(0) = 0, \quad \inf_{a \in \mathbf{R}_+} c_t''(a) > 0. \tag{2}$$

The principal's payoff, on the other hand, is a function of both agents' efforts over the two stages: $V(a_1^1, a_1^2, a_2^1, a_2^2)$. The function $V : \mathbf{R}_+^4 \to \mathbf{R}$ is assumed to be increasing ($V(\hat{a}) \geq V(a)$ if $\hat{a}_t^i \geq a_t^i$ for each t, $i = 1, 2$), and symmetric with respect to the agents ($V(\hat{a}) = V(a)$ if $\hat{a}_t^1 = a_t^2$ and $\hat{a}_t^2 = a_t^1$ for $t = 1, 2$). Furthermore, we assume that when the stage 2 efforts are symmetric $a_2^1 = a_2^2 = u$, $V(a_1, a_2)$ is a linear increasing function of u. In other words, there exist functions $A : \mathbf{R}_+^2 \to \mathbf{R}_+$ and $B : \mathbf{R}_+^2 \to \mathbf{R}$ such that for any a_1 and u,

$$V(a_1, a_2 = (u, u)) = A(a_1)u + B(a_1). \tag{3}$$

In essence, (3) ensures that when the stage 2 effort profile is symmetric, the principal cares only about their expected values. When the payoff function is time separable, for example, then (3) holds when the stage 2 payoff is homogeneous of degree one in the agents' stage 2 efforts. Leading examples of this case include

$$V(a) = \sum_{t=1}^{2} (a_t^1 + a_t^2), \quad V(a) = \sum_{t=1}^{2} \min\{a_t^1,\, a_t^2\},$$

and more generally, the CES family $V(a) = \sum_t \left\{(a_t^1)^m + (a_t^2)^m\right\}^{1/m}$ ($m \leq 1$, $m \neq 0$). Since the principal's payoff may contain more information about the agents' efforts than the scores x_t, we suppose that the principal observes his payoff after the winner has been determined so as to be in line with our assumption that the winner of the tournament is determined only by x.

Each agent's effort a_t^i is his private information and observed by neither the principal nor the other agent. On the other hand, the principal *privately* observes the score x_t in each stage t and reveals either whole or part of his private information x_1 after stage 1. Specifically, suppose that the principal makes a public announcement y about x_1 at the end of stage 1. Formally, a *feedback policy* (or simply a *policy*) is a pair of the set of possible announcements Y, and a measurable mapping $f : \mathbf{R} \to Y$, which chooses the announcement $y = f(x_1)$ as a function of the score x_1. For simplicity, the reference to Y will be omitted and the mapping f alone will be called a feedback policy in what follows. It is understood that $Y = \{f(x_1) : x_1 \in \mathbf{R}\}$ so that f is a surjection. The announcement y is *credible* in the sense that the principal publicly announces his feedback policy f before the tournament begins and uses it to generate y for any signal x_1. The principal's objective is to maximize his expected payoff by controlling f. Although we will restrict our analysis to deterministic feedback policies, the paper's conclusions hold even when we allow for stochastic feedback policies, which choose the announcement y as a function of x_1 *and* some (exogenous) random variable.

Little restriction is placed on the nature of the public announcement y. For example, each announcement $y \in Y$ may simply contain the name of the leader, or it may be an interval in $\mathbf{R}$ which indicates the range of x_1.

As mentioned in the Introduction, some simple feedback policies will play an import role in our analysis. In particular, the *no-feedback* policy sends the same message regardless of x_1, and the *full-feedback* policy reveals x_1 completely. Between these two are numerous policies that reveal an intermediate amount of information. For example, we will later discuss the hybrid policies which reveal full information when the score is within some range $(-b, b)$ $(b > 0)$, but nothing otherwise: $Y = (-b, b) \cup \{N\}$, and

$$f(x_1) = \begin{cases} x_1 & \text{if } |x_1| < b, \\ N & \text{otherwise.} \end{cases}$$

Of course, the agents hearing the announcement N under this policy would know that $|x_1| \geq b$. Given any announcement $y \in Y$, let $f^{-1}(y) = \{x_1 \in \mathbf{R} : f(x_1) = y\}$ denote the inverse image of the (singleton) set $\{y\}$ under f. In what follows, we will restrict attention to feedback policies that satisfy the following regularity condition: A feedback policy f is *regular* if for any $y \in Y$, $f^{-1}(y) \subset \mathbf{R}$ either has positive (Lebesgue) measure, or is countable. The above hybrid policy, for example, is regular since $f^{-1}(N) = (-\infty, -b) \cup (b, \infty)$ has positive measure and for $x_1 \in (-b, b)$, $f^{-1}(x_1) = \{x_1\}$ is countable.[17]

Given any policy f, agent i's *history* h^i after stage 1 is the information available to agent i at the end of stage 1: h^i consists of his own effort choice a_1^i, and the public announcement y by the principal. Agent i's (pure) *strategy* σ^i is a pair (σ_1^i, σ_2^i), where $\sigma_1^i \in \mathbf{R}_+$ is the effort choice for stage 1, and $\sigma_2^i : \mathbf{R}_+ \times Y \to \mathbf{R}_+$ is a mapping that specifies the stage 2 effort after each possible history $h^i = (a_1^i, y)$. Given the strategy profile σ, let $\pi_2^i(a_2^i \mid \sigma, h_1^i)$ denote agent i's expected payoff in stage 2 (payoff from the possible prize minus the cost of stage 2 effort) when he chooses a_2^i in stage 2, his history in stage 1 is h_1^i, and agent j plays according to the

[17] Feedback policy f fails to be regular if $f^{-1}(y)$ is, for example, the Cantor set for some y.

strategy σ^j in both stages. Likewise, let $\pi_1^i(a_1^i \mid \sigma)$ denote agent i's overall expected payoff when he chooses a_1^i in stage 1 and plays according to σ_2^i in stage 2, and agent j plays according to σ^j in both stages. Throughout, we consider an equilibrium of this tournament game in which each agent's effort choice is sequentially rational. In other words, the stage 2 effort choice of each agent i maximizes his expected payoff not only when his stage 1 effort choice is at the equilibrium level σ_1^i, but also when it is at any other level a_1^i. We also define an equilibrium simply in terms of a strategy profile since an agents' belief is unambiguously determined because of the assumption of full support of the distribution ϕ_1. Specifically, a strategy profile $\sigma = (\sigma^1, \sigma^2)$ is a (pure) *perfect Bayesian equilibrium* (PBE) if for $i = 1, 2$,

$$\pi_1^i(\sigma_1^i \mid \sigma) \geq \pi_1^i(a_1^i \mid \sigma) \text{ for any } a_1^i \in \mathbf{R}_+,$$

and

$$\pi_2^i(\sigma_2^i(h_1^i) \mid \sigma, h_1^i) \geq \pi_2^i(a_2^i \mid \sigma, h_1^i) \text{ for any } a_2^i \in \mathbf{R}_+ \text{ and } h_1^i \in \mathbf{R}_+ \times Y.$$

For any strategy $\sigma^i = (\sigma_1^i, \sigma_2^i)$ of agent i and announcement $y \in Y$, we define

$$\sigma_{2,0}^i(y) = \sigma_2^i(\sigma_1^i, y)$$

to be i's stage 2 effort given announcement y when he follows the strategy σ^i in both stages. If σ is an equilibrium, hence, $\sigma_{2,0}^i(y)$ denotes i's equilibrium effort choice in stage 2 given y. Given any strategy profile σ, stage 1 effort choice a_1^i of agent i, and public announcement y, $E^{\sigma,f}[\cdot \mid a_1^i, y]$ denotes the conditional expectation with respect to the stage 1 score x_1 given y when agent i chooses action a_1^i while agent j chooses σ_1^j in stage 1. Likewise, $E^{\sigma,f}[\cdot \mid y] = E^{\sigma,f}[\cdot \mid \sigma_1^i, y]$ denotes the conditional expectation given y when the stage 1 effort profile is $\sigma_1 = (\sigma_1^1, \sigma_1^2)$. The unconditional (ex ante) expectations $E^{\sigma,f}[\cdot \mid a_1^i]$ and $E^{\sigma,f}[\cdot]$ are defined in a similar manner.

Let $v(\sigma, f)$ denote the principal's expected payoff in a PBE σ under the feedback policy f:

$$v(\sigma, f) = E^{\sigma,f}\left[V\left(\sigma_1, \sigma_{2,0}(\tilde{y})\right)\right].$$

The principal's objective is to maximize $v(\sigma, f)$ by choosing a feedback policy f and inducing a PBE σ under f.

3 Equilibrium Effort Levels

We begin by deriving the essential marginal equation in stage 2. When the stage 1 score is x_1 and the stage 2 effort profile is (a_2^1, a_2^2), the probability that agent 1 wins is given by

$$1 - \Phi_2(-x_1 - a_2^1 + a_2^2) = \Phi_2(x_1 + a_2^1 - a_2^2),$$

where the equality follows from the symmetry of ϕ_2 around 0: $\Phi_2(x) = 1 - \Phi_2(-x)$ for any x. Hence, given the history $h_1^1 = (a_1^1, y)$ of his own action and public announcement, agent 1's expected payoff in stage 2 can be written as:

$$\pi_2^1(a_2^1 \mid \sigma, h_1^1) = E^{\sigma,f}[\Phi_2(\tilde{x}_1 + a_2^1 - \sigma_{2,0}^2(y)) \mid a_1^1, y] - c_2(a_2^1).$$

Taking the derivative with respect to a_2^1, we see that the sequentially rational choice of effort $a_2^1 = \sigma_2^1(a_1^1, y)$ in stage 2 should satisfy the first-order condition

$$E^{\sigma,f}[\phi_2(\tilde{x}_1 + \sigma_2^1(a_1^1, y) - \sigma_{2,0}^2(y)) \mid a_1^1, y] = c_2'(\sigma_2^1(a_1^1, y)).$$

When a_1^1 equals the equilibrium effort choice σ_1^1 in stage 1, hence, the effort choice $\sigma_{2,0}^1(y) = \sigma_2^1(\sigma_1^1, y)$ on the path of play in stage 2 should satisfy

$$E^{\sigma,f}[\phi_2(\tilde{x}_1 + \sigma_{2,0}^1(y) - \sigma_{2,0}^2(y)) \mid y] = c_2'(\sigma_{2,0}^1(y)). \tag{4}$$

This is the marginal equation for agent 1 in stage 2: He balances the expected marginal increment in the probability of winning with the marginal disutility of effort. The corresponding condition for agent 2 is given by

$$E^{\sigma,f}[\phi_2(-\tilde{x}_1 - \sigma_{2,0}^1(y) + \sigma_{2,0}^2(y)) \mid y] = c_2'(\sigma_{2,0}^2(y)). \tag{5}$$

With the symmetry of ϕ_2, the expected marginal increments on the left-hand sides of (4) and (5) are indeed the same, and so are $\sigma_{2,0}^1(y)$ and $\sigma_{2,0}^2(y)$. With the two agents' efforts canceling each other, the expected marginal increment reduces to $E^{\sigma,f}[\phi_2(\tilde{x}_1) \mid y]$ in equilibrium. The following theorem summarizes this argument and also describes the first-order conditions for the stage 1 effort in any pure PBE. Recall that $\bar{\phi}$ denotes the density of the aggregate noise $\zeta_1 + \zeta_2$.

Theorem 3.1. *Suppose that*

$$\sup_{x \in \mathbf{R}} \phi_2'(x) < \inf_{a \in \mathbf{R}_+} c_2''(a).$$

If σ is a pure PBE under any feedback policy f, then for any $y \in Y$,

$$\sigma_{2,0}^1(y) = \sigma_{2,0}^2(y) = \alpha_2(\sigma_1, y) \equiv (c_2')^{-1}\left(E^{\sigma,f}[\phi_2(\tilde{x}_1) \mid y]\right). \tag{6}$$

If, in addition, σ_1^1, $\sigma_1^2 > 0$, then

$$\begin{cases} c_1'(\sigma_1^1) = \bar{\phi}(\sigma_1^1 - \sigma_1^2) + \int_{\mathbf{R}} c_2\left(\alpha_2(\sigma_1, f(x_1))\right) \phi_1'(x_1 - \sigma_1^1 + \sigma_1^2)\, dx_1, \\ c_1'(\sigma_1^2) = \bar{\phi}(\sigma_1^1 - \sigma_1^2) - \int_{\mathbf{R}} c_2\left(\alpha_2(\sigma_1, f(x_1))\right) \phi_1'(x_1 - \sigma_1^1 + \sigma_1^2)\, dx_1. \end{cases} \tag{7}$$

Proof. See the Appendix. ■

As seen above, the stage 2 effort is determined through the standard marginal consideration, and the symmetry of the agents' stage 2 effort profile holds for any announcement whether the equilibrium itself is symmetric or not. It can also be seen that the expected marginal cost in stage 2 is independent of the feedback policy since by the law of iterated expectation

$$\begin{aligned} E^{\sigma,f}\left[c_2'(\alpha_2(\sigma_1, \tilde{y}))\right] &= E^{\sigma,f}\left[E^{\sigma,f}\left[\phi_2(\tilde{x}_1) \mid \tilde{y}\right]\right] \\ &= E^{\sigma,f}\left[\phi_2(\tilde{x}_1)\right] \\ &= \bar{\phi}(0). \end{aligned} \tag{8}$$

One implication of (6) is as follows. Suppose for simplicity that f is the full-feedback policy: $f(x_1) = x_1$. In this case, $\sigma_{2,0}^i(x_1) = (c_2')^{-1}\left(\phi_2(x_1)\right)$ as is readily verified. It follows that the stage 2 effort is maximized when $\phi_2(x_1)$ is the largest. If ϕ_2 is unimodal at the origin as in the case of the normal distribution, hence, the stage 2 effort is a monotone decreasing function of $|x_1|$. This supports the common intuition that the closer the competition, the higher the efforts the agents exert. Note, however, that this intuition fails when, for example, ϕ_2 is bimodal so that $\phi_2(x) = \phi_2(-x) > \phi_2(0)$ for some $x > 0$.

We next turn to the existence of a pure PBE. Let

$$\kappa = \min\left\{1, \inf_{a \in \mathbf{R}_+} c_1''(a), \inf_{a \in \mathbf{R}_+} c_2''(a), \lim_{a \to \infty} c_2'(a)\right\} > 0. \tag{9}$$

The next theorem identifies a sufficient condition for the existence.

Theorem 3.2. *Suppose that*

$$\sup_{x \in \mathbf{R}} \phi_2(x),\ \sup_{x \in \mathbf{R}} \phi_2'(x),\ \int_{\mathbf{R}} |\phi_1''(x)|\, dx,\ \int_{\mathbf{R}} \frac{\phi_1'(x)^2}{\phi_1(x)}\, dx < \frac{\kappa}{2} \tag{10}$$

where κ is as defined in (9). Given any feedback policy f, there exists a pure PBE under f if (7) has a solution $\sigma_1 = (\sigma_1^1, \sigma_1^2) \geq 0$.

Proof. See the Appendix. ■

The conditions involving ϕ_t in Theorems 1 and 2 amount to requiring that the performance score be a sufficiently noisy signal of agents' effort. If, for example, ζ_t has the normal distribution $N(0, \sigma_t^2)$ $(t = 1, 2)$, then these conditions are satisfied if the variances σ_1^2 and σ_2^2 are sufficiently large.[18] As discussed in Nalebuff and Stiglitz (1983), large noise is a standard requirement for the existence of an equilibrium in a tournament model with stochastic performance. Intuitively, if the noise is too small, then any infinitesimal increase in effort results in almost sure winning, making it impossible for the marginal equation to hold. It should also be emphasized that in Theorem 2, the noise level required for the existence of an equilibrium is independent of a particular feedback policy f.

In what follows, we assume for simplicity that Y is a vector space and normalize $f(0) = 0 \in Y$. With this standardization, we say that a feedback policy f is *odd* if $f(x) = -f(-x)$ for any $x \in \mathbf{R}$ and *even* if $f(x) = f(-x)$ for any $x \in \mathbf{R}$. Intuitively, if f is odd, then the inference drawn from the announcement when agent i leads agent j in stage 1 is the exact opposite of that when their positions are reversed. On the other hand, if f is even, then the announcement is the same regardless of the identity of the leader as long as the size of the lead is the same. For example, the full-feedback policy $f(x) = x$ is odd (but not even), whereas the no-feedback policy $f(x) \equiv 0$ is the only policy that is both odd and even.

A strategy profile σ is *symmetric* if the two agents always choose the same effort level on the path: $\sigma_1^1 = \sigma_1^2$ and $\sigma_{2,0}^1(y) = \sigma_{2,0}^2(y)$ for any $y \in Y$. We now show that every even or odd policy admits a symmetric PBE when the noise is sufficiently large. By summing the two equations of (7), we see that the stage 1 effort in a symmetric PBE (if any) must satisfy

$$\sigma_1^1 = \sigma_1^2 = a_1^* \equiv (c_1')^{-1}(\bar{\phi}(0)). \tag{11}$$

The following theorem confirms that this a_1^* is indeed part of an equilibrium when f is even or odd.

[18] In fact, note that $\sup_{x\in\mathbf{R}} \phi_2(x) = \frac{1}{\sqrt{2\pi}\sigma_2}$, $\sup_{x\in\mathbf{R}} \phi_2'(x) = \frac{1}{\sqrt{2\pi}\,\sigma_2^2}\, e^{-1/2}$,

$$\int_{\mathbf{R}} |\phi_1''(x)|\, dx = \frac{1}{\sqrt{2\pi}\sigma_1} \int_{\mathbf{R}} \left| -\frac{1}{\sigma_1^2}\, e^{-x^2/2\sigma_1^2} + \frac{x^2}{\sigma_1^4}\, e^{-x^2/2\sigma_1^2} \right| dx \le \frac{2}{\sigma_1^2},$$

and

$$\int_{\mathbf{R}} \frac{\phi_1'(x)^2}{\phi_1(x)}\, dx = \frac{1}{\sigma_1^2}.$$

Theorem 3.3. *Suppose that condition (10) hold. If f is either odd or even, there exists a unique symmetric pure PBE σ. Furthermore, for α_2 defined in (6) and a_1^* defined in (11), σ satisfies*

$$\sigma_1^1 = \sigma_1^2 = a_1^*, \tag{12}$$

and

$$\sigma_{2,0}^1(y) = \sigma_{2,0}^2(y) = \alpha_2(\sigma_1, y) \quad \text{for any } y \in Y.$$

Proof. See the Appendix. ■

The following comments are in order on Theorem 3.3. First, it can be seen from (8) and (12) that in symmetric PBE, the marginal cost of effort in stage 1 and the expected marginal cost in stage 2 both equal $\bar{\phi}(0)$. This is a very intuitive intertemporal relationship whose counterpart forms the basis of, for example, the theory of precautionary savings. As mentioned in the Introduction, however, this intertemporal equality holds only for a symmetric PBE in which the strategic effects of both agents offset each other. Second, the stage 1 effort in symmetric PBE is independent of the choice of a feedback policy. It is possible to interpret this fact through the above intertemporal equality: As seen in (8), the expected stage 2 marginal cost is independent of the choice of a feedback policy, and so is the marginal cost in stage 1 by virtue of the intertemporal equality. This shows that the stage 1 effort itself is the same in symmetric PBE under any policy. The following facts about the no-feedback and full-feedback policies are readily implied by Theorem 3.3.

Proposition 3.4. *If σ is the (unique) symmetric pure PBE under the no-feedback policy, then the stage 1 effort equals $\sigma_1^i = a_1^*$ and the stage 2 effort equals*

$$\sigma_{2,0}^i = a_2^N \equiv (c_2')^{-1}\left(\bar{\phi}(0)\right).$$

Likewise, if σ is the (unique) symmetric pure PBE under the full-feedback policy, then the stage 1 effort equals $\sigma_1^i = a_1^*$ and the expected stage 2 effort equals

$$E^{\sigma,f}[\sigma_{2,0}^i(\tilde{y})] = a_2^F \equiv \int_{\mathbf{R}} (c_2')^{-1}(\phi_2(x_1))\,\phi_1(x_1)\,dx_1.$$

When $(c_2')^{-1}$ is concave or convex, Proposition 4 can be used to rank the no-feedback and full-feedback policies in terms of the expected stage 2 effort they induce in the symmetric PBE. Suppose for example that $(c_2')^{-1}$ is concave. Then Jensen's inequality implies the no-feedback policy induces a higher expected effort than the full-feedback policy:

$$a_2^F = \int_{\mathbf{R}} (c_2')^{-1}(\phi_2(x_1))\,\phi_1(x_1)\,dx_1 \le (c_2')^{-1}\left(\int_{\mathbf{R}} \phi_2(x_1)\,\phi_1(x_1)\,dx_1\right) = a_2^N.$$

The reverse inequality holds when $(c_2')^{-1}$ is convex. In the next section, we consider generalizations of these inequalities.

4 Optimal Feedback Policy

In this section, we will study the principal's expected payoff in the pure PBE as identified in Theorems 3.1-3.3. Given that the stage 2 efforts in any PBE are always symmetric between the two agents by (6), it follows from our assumption (3) on the principal's payoff function V that his expected payoff is an increasing function of their expected stage 2 effort.[19] Recall that $v(\sigma, f)$ denotes the principal's expected payoff in a PBE σ under the feedback policy f.

4.1 Symmetric Equilibrium

Even when f admits multiple symmetric pure PBE's, they all induce the same on-the-path effort by Theorem 3.1 and equation (11). In this sense, the principal's payoff is independent of the choice of a symmetric PBE σ. Hence, we define

$$\bar{v}^*(f) = \begin{cases} v(\sigma, f) & \text{if } f \text{ admits a symmetric pure PBE } \sigma, \\ 0 & \text{otherwise.} \end{cases}$$

Theorem 4.1. *Suppose that condition (10) holds. If the marginal cost function c_2' for stage 2 is convex over $[0, (c_2')^{-1}(\sup_{x\in\mathbf{R}} \phi_2(x))]$, then the no-feedback policy maximizes $\bar{v}^*$ among all policies.*

Proof. Take any policy f with a symmetric pure PBE σ. By the preceding discussions, it suffices to show that the expected stage 2 effort is maximized under the

[19] In the consideration of a symmetric PBE in Section 4.1, we only need the linearity of $V(a_1, a_2^1 = a_2^2 = u)$ in u for a_1 such that $a_1^1 = a_1^2$.

no-feedback policy. Since $(c_2')^{-1}$ is concave over $[0, \sup_{x\in\mathbf{R}} \phi_2(x)]$, it follows from Jensen's inequality and the law of iterated expectation that the expected stage 2 effort under f satisfies

$$\begin{aligned} E^{\sigma,f}[\alpha_2(\sigma_1,\tilde{y})] &= E^{\sigma,f}\left[(c_2')^{-1}\left(E^{\sigma,f}[\phi_2(\tilde{x}_1) \mid \tilde{y}]\right)\right] \\ &\le (c_2')^{-1}\left(E^{\sigma,f}\left[E^{\sigma,f}[\phi_2(\tilde{x}_1) \mid \tilde{y}]\right]\right) \\ &= (c_2')^{-1}\left(E^{\sigma,f}\left[\phi_2(\tilde{x}_1)\right]\right) \\ &= (c_2')^{-1}\left(\bar{\phi}(0)\right) \\ &= a_2^N, \end{aligned}$$

where the third equality follows from the symmetry of the stage 1 effort profile σ_1. ■

Theorem 4.2. *Suppose that condition (10) holds. If the marginal cost function c_2' for stage 2 is concave over $[0, (c_2')^{-1}(\sup_{x\in\mathbf{R}} \phi_2(x))]$, then the full-feedback policy maximizes $\bar{v}^*$ among all policies.*

Proof. Take any policy f with a symmetric PBE σ. By the same logic as above, it suffices to show that the expected stage 2 effort is maximized under the full-feedback policy. Since $(c_2')^{-1}$ is convex over $[0, \sup_{x\in\mathbf{R}} \phi_2(x)]$, we now have

$$\begin{aligned} E^{\sigma,f}[\alpha_2(\sigma_1,\tilde{y})] &= E^{\sigma,f}\left[(c_2')^{-1}\left(E^{\sigma,f}[\phi_2(\tilde{x}_1) \mid \tilde{y}]\right)\right] \\ &\le E^{\sigma,f}\left[E^{\sigma,f}\left[(c_2')^{-1}\left(\phi_2(\tilde{x}_1)\right) \mid \tilde{y}\right]\right] \\ &= E^{\sigma,f}\left[(c_2')^{-1}\left(\phi_2(\tilde{x}_1)\right)\right] \\ &= \int_{\mathbf{R}} (c_2')^{-1}(\phi_2(x_1))\, \phi_1(x_1)\, dx_1 \\ &= a_2^F. \end{aligned}$$

where again the third equality follows from the symmetry of the stage 1 effort profile σ_1. ■

The proofs of Theorems 4.1 and 4.2 also indicate that when c_2' is concave (resp. convex), the no-feedback (resp. full-feedback) policy yields the *lowest* expected payoff to the principal. On the other hand, when the marginal cost function c_2' for stage 2 is linear (and hence both concave and convex), the induced effort in either stage is not affected by the feedback policy. The following corollary summarizes this immediate consequence of Theorem 3.1.

Corollary 4.3. *Suppose that the stage 2 cost function is quadratic:* $c_2(a) = \frac{1}{2}\, ka^2$ *for some* $k > 0$*. Suppose also that* $\sup_{x\in\mathbf{R}} \phi_2'(x) < k$*. Let* f *be any feedback policy. In any symmetric pure PBE* σ *under* f*, the stage 1 effort equals* $\sigma_1^i = a_1^*$*, and the expected stage 2 effort equals* $E^{\sigma,f}[\sigma_{2,0}^i(\tilde{y})] = \frac{1}{k}\,\bar{\phi}(0)$*. It follows that the principal's expected payoff* $v(\sigma, f)$ *is independent of* f*.*

4.2 Asymmetric Equilibrium

We now allow a PBE σ to be asymmetric, and define

$$\bar{v}(f) = \sup\big\{v(\sigma, f) : \sigma \text{ is a pure PBE under } f \text{ and satisfies (7)}\big\},$$

with $\bar{v}(f) = -\infty$ if the corresponding strategy profile does not exist. We will make some additional assumptions in order to evaluate the principal's expected payoff when the stage 1 effort profile is asymmetric. Specifically, we will identify the situations where the principal obtains a higher payoff in a symmetric PBE than in an asymmetric PBE. In such situations, the optimality of the no-feedback or full-feedback policies is obtained just as before. Intuitively, a symmetric PBE is more desirable for the principal than an asymmetric PBE if the two agents' efforts enter his payoff function in a complementary manner. In other words, we would want the principal's payoff to be higher when both agents make moderate efforts than when one agent makes a high effort and the other makes a low effort. The assumption below specifies just how much complementarity is sufficient for our conclusion.

Assumption 1. *The principal's payoff function* V *is differentiable, and for any* $a = \big((a_1^1, a_1^2), (a_2^1, a_2^2)\big) \in \mathbf{R}_+^4$ *such that* $a_1^1 < a_1^2$ *and* $a_2^1 = a_2^2$*, we have*

$$\frac{c_1''(a_1^1) - 2\bar{\phi}'(a_1^1 - a_1^2)}{c_1''(a_1^2) + 2\bar{\phi}'(a_1^1 - a_1^2)} < \frac{\frac{\partial V}{\partial a_1^1}(a)}{\frac{\partial V}{\partial a_1^2}(a)}. \tag{13}$$

It can be seen that the left-hand side of (13) represents the slope of the curve

$$h(a_1^1, a_1^2) \equiv c_1'(a_1^1) + c_1'(a_1^2) - 2\bar{\phi}(a_1^1 - a_1^2) = 0 \tag{14}$$

in the (a_1^1, a_1^2)-plane, which equals the sum of the two first-order conditions in (7). On the other hand, the right-hand side of (13) represents the slope of the principal's iso-payoff curve. Hence, (13) is a single-crossing condition asserting that the iso-payoff curve always has a steeper slope than (14). To see that this implies complementarity between the two agents' efforts, suppose that V has the CES form: $V(a) = \sum_t \left\{(a_t^1)^m + (a_t^2)^m\right\}^{1/m}$, where $m \leq 1$ and $m \neq 0$. In this case, the right-hand side of (13) equals $(a_1^1/a_1^2)^{m-1}$. Hence, (13) is easy to satisfy when $m-1$ is negative and large in absolute value. In particular, it will hold for any c_1 as $m \to -\infty$, or $V(a) = \sum_t \min\{a_t^1, a_t^2\}$ in the limit. On the other hand, in the case of perfect substitutes $m = 1$, the inequality reduces to $c_1''(a_1^1) - c_1''(a_1^2) < 4\bar{\phi}'(a_1^1 - a_1^2)$, which in effect requires c_1''' to be not too negative. The second assumption below requires that the density of the aggregate noise be maximized at the origin.

Assumption 2. $\bar{\phi}(0) = \max_{x \in \mathbf{R}} \bar{\phi}(x)$.

It can be readily verified that Assumption 2 holds if both densities ϕ_1 and ϕ_2 are unimodal at the origin. As seen in the Appendix (Lemma A.1), Assumptions 1 and 2 together guarantee that the principal's payoff is maximized at the symmetric point (a_1^*, a_1^*) along (14) (provided that the stage 2 efforts are symmetric). The next theorem shows that when these conditions hold, the principal's payoff is maximized in the symmetric equilibrium under the no-feedback policy if the stage 2 marginal cost function is convex.

Theorem 4.4. *Suppose that Assumptions 1-2 and condition (10) hold. If the marginal cost function c_2' for stage 2 is convex over $[0, (c_2')^{-1}(\sup_{x \in \mathbf{R}} \phi_2(x))]$, then the no-feedback policy maximizes $\bar{v}(\cdot)$ among all policies.*

Proof. See the Appendix. ■

For the other type of the conclusion, we also need the density function of the stage 2 noise to be unimodal.

Assumption 3. *ϕ_2 is unimodal at 0: ϕ_2 is strictly increasing over $(-\infty, 0)$ and strictly decreasing over $(0, \infty)$.*

Under Assumptions 1-3, we obtain the optimality of the full-feedback policy when the stage 2 marginal cost function is concave.

Theorem 4.5. *Suppose that Assumptions 1-3 and condition (10) hold. If the marginal cost function c_2' for stage 2 is concave over $[0, (c_2')^{-1}(\phi_2(0))]$, then the full-feedback policy maximizes $\bar{v}(\cdot)$ among all policies.*[20]

[20] Note that $\sup_{x \in \mathbf{R}} \phi_2(x) = \phi_2(0)$ under Assumption 3.

Proof. See the Appendix. ■

As in the case of symmetric PBE, the principal's payoff is independent of the choice of a feedback policy when the stage 2 cost function c_2 is quadratic.

4.3 Optimality of Intermediate Policies

Given the conclusions of the preceding sections, we now turn to the question of optimal policies when the marginal cost of effort is neither convex nor concave over the relevant domain. We are specifically interested in the existence of a feedback policy that induces a higher effort than the no-feedback and full-feedback policies for such a cost function. Among many different possibilities, we examine the simplest case where the stage 2 marginal cost function c_2' has a single reflection point at which the curvature changes from convex to concave. More specifically, we assume that the density for the stage 2 noise is unimodal at the origin (Assumption 3), and that c_2' satisfies the following condition.

Assumption 4. *There exists $r \in \big(0, (c_2')^{-1}(\phi_2(0))\big)$ such that the stage 2 marginal cost function c_2' is convex over $[0, r]$ and concave over $[r, (c_2')^{-1}(\phi_2(0))]$.*

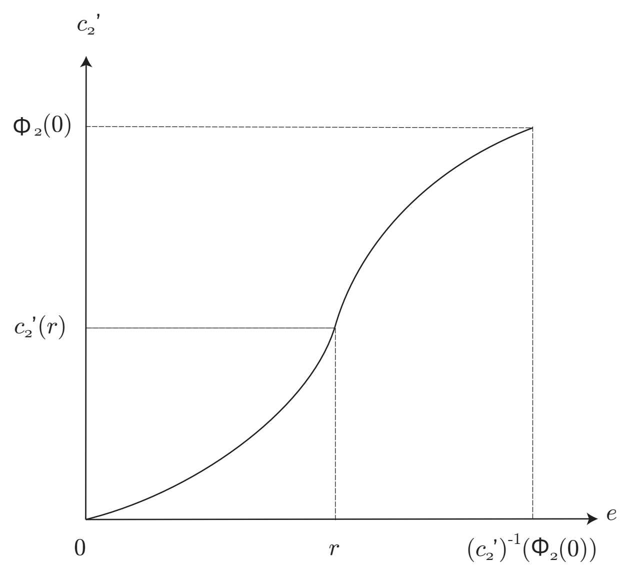

Figure 1

Figure 1 illustrates c_2' satisfying the above condition. Let $\rho > 0$ be such that $\phi_2(\rho) = c_2'(r)$. It follows that when the stage 1 score is such that $|x_1| < \rho$, then

$\phi_2(x_1) > c_2'(r)$ so that c_2' is concave at $(c_2')^{-1}(\phi_2(x_1))$, and likewise, when $|x_1| \geq \rho$, then $\phi_2(x_1) \leq c_2'(r)$ so that c_2' is convex at $(c_2')^{-1}(\phi_2(x_1))$. By the discussion in the previous sections, a natural focus is on the feedback policy that reveals full information if $x_1 \in (-\rho, \rho)$ and no information otherwise. Let this feedback policy be denoted by f^*:

$$f^*(x_1) = \begin{cases} x_1 & \text{if } |x_1| < \rho, \\ \rho & \text{otherwise.} \end{cases} \tag{15}$$

Note that f^* is even, and that the announcement ρ under f^* merely indicates the fact that $|x_1| \geq \rho$. The following theorem shows that f^* is optimal in a class of policies that include the full-feedback policy. Recall that $\bar{v}^*(f)$ denotes the principal's payoff in the symmetric PBE under f.

Theorem 4.6. *Suppose that Assumptions 3, 4 and condition (10) hold. Consider a class of feedback policies f which reveal whether $|x_1| < \rho$ or not, i.e., $f(x_1) \neq f(x_1')$ for any x_1 and x_1' such that $|x_1| < \rho$ and $|x_1'| \geq \rho$. Then f^* specified in (15) maximizes $\bar{v}^*$ in this class.*

Proof. Let σ be the symmetric PBE under any feedback policy f in such a class. We then have

$$|x_1| < \rho \implies E^{\sigma,f}[\phi_2(\tilde{x}_1) \mid \tilde{y} = f(x_1)] > c_2'(r), \quad \text{and}$$
$$|x_1| \geq \rho \implies E^{\sigma,f}[\phi_2(\tilde{x}_1) \mid \tilde{y} = f(x_1)] \leq c_2'(r)$$

It follows that the expected stage 2 effort under f satisfies

$$\begin{aligned} & E^{\sigma,f}\left[(c_2')^{-1}\left(E^{\sigma,f}[\phi_2(\tilde{x}_1) \mid \tilde{y}]\right)\right] \\ &= E^{\sigma,f}\left[(c_2')^{-1}\left(E^{\sigma,f}[\phi_2(\tilde{x}_1) \mid \tilde{y}]\right) \,\Big|\, |\tilde{x}_1| \geq \rho\right] P^{\sigma,f}\left(|\tilde{x}_1| \geq \rho\right) \\ &+ E^{\sigma,f}\left[(c_2')^{-1}\left(E^{\sigma,f}[\phi_2(\tilde{x}_1) \mid \tilde{y}]\right) \,\Big|\, |\tilde{x}_1| < \rho\right] P^{\sigma,f}\left(|\tilde{x}_1| < \rho\right) \\ &\leq (c_2')^{-1}\left(E^{\sigma,f}\left[E^{\sigma,f}[\phi_2(\tilde{x}_1) \mid \tilde{y}] \,\Big|\, |\tilde{x}_1| \geq \rho\right]\right) P^{\sigma,f}\left(|\tilde{x}_1| \geq \rho\right) \\ &+ E^{\sigma,f}\left[E^{\sigma,f}\left[(c_2')^{-1}(\phi_2(\tilde{x}_1)) \mid \tilde{y}\right] \,\Big|\, |\tilde{x}_1| < \rho\right] P^{\sigma,f}\left(|\tilde{x}_1| < \rho\right) \\ &= (c_2')^{-1}\left(E^{\sigma,f}[\phi_2(\tilde{x}_1) \mid |\tilde{x}_1| \geq \rho]\right) P^{\sigma,f}\left(|\tilde{x}_1| \geq \rho\right) \\ &+ E^{\sigma,f}\left[(c_2')^{-1}(\phi_2(\tilde{x}_1)) \mid |\tilde{x}_1| < \rho\right] P^{\sigma,f}\left(|\tilde{x}_1| < \rho\right), \end{aligned} \tag{16}$$

where the inequality follows from the above observation as well as Jensen's inequality, and the last equality from the fact that the filtration induced by the announcement $\tilde{y}$ includes the events $\{|\tilde{x}_1| \geq \rho\}$ and $\{|\tilde{x}_1| < \rho\}$. Since the far right-hand side of (16) equals the expected stage 2 effort under f^*, the desired conclusion follows. ■

Since the full-feedback policy certainly distinguishes the two events $|x_1| \leq \rho$ and $|x_1| > \rho$, the above theorem readily implies the following corollary.

Corollary 4.7. *Suppose that Assumptions 3, 4 and condition (10) hold. Then f^* specified in (15) yields the principal a higher expected payoff than the full feedback policy.*

The proof of Theorem 4.6 also suggests that any policy f measurable with respect to some subset of $\{x_1 : |x_1| < \rho\}$ or $\{x_1 : |x_1| \geq \rho\}$ is dominated by another policy. To be more precise, let $B \subset \{x_1 : |x_1| < \rho\}$ and suppose that f is measurable with respect to B, *i.e.*, $f(x_1) \neq f(x_1')$ for any x_1 and x_1' such that $x_1 \in B$ and $x_1' \notin B$. Then a slight modification of the above proof shows that f is (weakly) dominated by an alternative policy that reveals full information when $x_1 \in B$, but is the same as f when $x_1 \notin B$. The following theorem extends this kind of logic further to give a sufficient condition for the no-feedback policy to be suboptimal.

Theorem 4.8. *Suppose that Assumptions 3, 4 and condition (10) hold. If*

$$\bar{\phi}(0) > c_2'(r),$$

then there exists a feedback policy that induces a higher expected effort than the no-feedback policy.

Proof. See the Appendix. ■

The proof of the above theorem shows that the no-feedback policy is dominated by a policy which only reveals whether or not $\phi_2(x_1)$ has exceeded a certain threshold.

5 Private Feedback

Our analysis has so far assumed that the agents' information at the beginning of stage 2 consists only of his own action in stage 1 and feedback from the principal about the stage 1 score. As mentioned in the Introduction, however, it may be more appropriate in some applications to suppose that the agents privately observe their own performance even with no information feedback. In this section, we model such a situation and examine how revealing additional information affects the agents' effort incentives. Interestingly, it can be shown that there still exists the same monotone relationship as before between the amount of information revealed and the induced expected effort. Specifically, when the stage 2 marginal cost function is concave, an agent's expected effort is higher if he is informed of the other agent's stage 1 performance. If the marginal cost is convex, his expected effort is higher when no such information is provided.

It should be noted that private observation of one's own performance can be interpreted in two ways. In one interpretation, the agents inherently know their own performance as would be the case when there is an objective measure for performance evaluation. In the other interpretation, the agents are initially ignorant (as in the original setup) but privately learn their own performance as a result of private feedback of such information form the principal. The two interpretations are formally equivalent. We adopt the second interpretation in what follows and refer to the situation where each agent is informed only of his own performance as *private feedback.* A version of private feedback can also be found in Mares and Harstad (2002), who show in a common-value auction setting that an auctioneer may be better off revealing his private information in a non-public way.

Unlike in the preceding sections, we now suppose that each agent's effort in stage 1 gives rise to his individual performance. Formally, agent i's *performance* z_1^i in stage 1 is the sum of his effort a_1^i and an exogenous random variable ε_1^i: $z_1^i = a_1^i + \varepsilon_1^i$. The stage 1 score x_1 is the difference between the two performance levels: $x_1 = a_1^1 - a_1^2 + \varepsilon_1^1 - \varepsilon_1^2$. Just as before, the performance score in stage 2 is generated according to $x_2 = a_2^1 - a_2^2 + \zeta_2$, and the winner is determined by the aggregate score $x = x_1 + x_2$. With private feedback, the agents are asymmetrically informed about their performance, and hence may choose an asymmetric effort profile in stage 2. This is a significant departure from the preceding analysis and creates much difficulty. For this reason, we make the following simplifying assumptions on the distributions of noise. First, the joint distribution of the stage 1 noise profile $(\varepsilon_1^1, \varepsilon_1^2)$ is finite and given by

$$
\begin{aligned}
&P(\tilde{\varepsilon}_1^1 = \tilde{\varepsilon}_1^2 = 0) = \alpha, \quad P(\tilde{\varepsilon}_1^1 = \tilde{\varepsilon}_1^2 = q) = \beta, \\
&P(\tilde{\varepsilon}_1^1 = 0,\, \tilde{\varepsilon}_1^2 = q) = P(\tilde{\varepsilon}_1^1 = q,\, \tilde{\varepsilon}_1^2 = 0) = \gamma,
\end{aligned} \tag{17}
$$

where $q, \alpha, \beta, \gamma > 0$ are constants and satisfy $2\gamma = 1 - \alpha - \beta$. Second, the density function ϕ_2 of the stage 2 noise ζ_2 has bounded support such that

$$
\phi_2(x) = 0 \text{ if } |x| \geq q - (c_1)^{-1}(1) - (c_2)^{-1}(1). \tag{18}
$$

Intuitively, this assumption states that q is large and represents an *insurmountable gap* in performance levels. In other words, when $\varepsilon_1^i = q$ and $\varepsilon_1^j = 0$, agent i becomes the ultimate winner as long as neither agent in any stage chooses an effort whose cost exceeds the value of winning. This assumption simplifies the analysis since then we can focus on the effort levels that follow the noise profile $(\varepsilon_1^1, \varepsilon_1^2)$ such that $\varepsilon_1^1 = \varepsilon_1^2$. As in the previous sections, we also need to require the outcome to be sufficiently noisy in order to guarantee the sufficiency of the first-order conditions for maximization as well as the existence of an equilibrium. Specifically, we assume throughout this section that

$$\text{(19)} \qquad \sup \phi_2'(x) < \min\left\{\frac{1}{\kappa}\inf c_2''(a), \left(1-\frac{1}{\kappa}\right)\inf c_1''(a)\right\} \quad \text{for some } \kappa > 1.$$

As mentioned above, we suppose that the agents initially do not know their own stage 1 performance, but that the principal can use the private feedback policy that informs each agent i of his own performance z_1^i. Given the binary nature of the noise distribution for each agent, we will consider the no-feedback and full-feedback policies as alternatives. Our main objective is to compare the agents' expected effort under these three policies. As seen in Proposition 5.1, it can be shown that there exists a unique PBE outcome under each one of these policies, and that the equilibrium effort is characterized by the same marginal equation.

In what follows, we illustrate the derivation of a PBE σ under the private feedback policy. Note first that for agent i, learning $z_1^i = a_1^i + \varepsilon_1^i$ is equivalent to observing the noise term ε_1^i. Given his stage 1 effort choice a_1^1 and noise realization ε_1^1, agent 1's stage 2 payoff function is given by

$$\begin{aligned}&\pi_2^1(a_2^1 \mid a_1^1, \varepsilon_1^1)\\ &= P(\tilde{\varepsilon}_1^2 = \varepsilon_1^1 \mid \varepsilon_1^1)\,\Phi_2\Big(a_1^1 - \sigma_1^2 + a_2^1 - \sigma_{2,0}^2(\varepsilon_1^1)\Big) + P(\tilde{\varepsilon}_1^2 < \varepsilon_1^1 \mid \varepsilon_1^1) - c_2(a_2^1).\end{aligned}$$

Hence, the sequentially rational choice of effort $\sigma_2^1(a_2^1, \varepsilon_1^1)$ satisfies the FOC

$$P(\tilde{\varepsilon}_1^2 = \varepsilon_1^1 \mid \varepsilon_1^1)\,\phi_2\Big(a_1^1 - \sigma_1^2 + \sigma_2^1(a_1^1, \varepsilon_1^1) - \sigma_{2,0}^2(\varepsilon_1^1)\Big) - c_2'(\sigma_2^1(a_1^1, \varepsilon_1^1)) = 0.$$

When the stage 1 effort a_1^1 is at the equilibrium level σ_1^1, agent 1's stage 2 effort choice $\sigma_{2,0}^1(\varepsilon_1^1) = \sigma_2^1(\sigma_1^1, \varepsilon_1^1)$ on the path satisfies

$$P(\tilde{\varepsilon}_1^2 = \varepsilon_1^1 \mid \varepsilon_1^1)\,\phi_2\Big(\sigma_1^1 - \sigma_1^2 + \sigma_{2,0}^1(\varepsilon_1^1) - \sigma_{2,0}^2(\varepsilon_1^1)\Big) = c_2'(\sigma_{2,0}^1(\varepsilon_1^1)).$$

Since for $j \neq i$,

$$P(\tilde{\varepsilon}_1^j = \varepsilon_1^i \mid \varepsilon_1^i) = \begin{cases} \frac{\alpha}{\alpha+\gamma} & \text{if } \varepsilon_1^i = 0,\\ \frac{\beta}{\beta+\gamma} & \text{if } \varepsilon_1^i = q,\end{cases}$$

the above FOC and the corresponding condition for agent 2 yield

$$\sigma_{2,0}^i(\varepsilon_1^i) = \begin{cases} (c_2')^{-1}(\frac{\alpha}{\alpha+\gamma}\phi_2(\sigma_1^1 - \sigma_1^2)) & \text{if } \varepsilon_1^i = 0, \\ (c_2')^{-1}(\frac{\beta}{\beta+\gamma}\phi_2(\sigma_1^1 - \sigma_1^2)) & \text{if } \varepsilon_1^i = q. \end{cases}$$

With the stage 2 effort choice specified, we can write agent 1's overall payoff as a function of his stage 1 effort a_1^1 as:

$$\begin{aligned} \pi_1^1(a_1^1) = {} & \alpha\,\Phi_2\Big(a_1^1 - \sigma_1^2 + \sigma_2^1(a_1^1, 0) - \sigma_{2,0}^2(0)\Big) - (\alpha+\gamma)\,c_2(\sigma_2^1(a_1^1, 0)) \\ & + \beta\,\Phi_2\Big(a_1^1 - \sigma_1^2 + \sigma_2^1(a_1^1, q) - \sigma_{2,0}^2(q)\Big) - (\beta+\gamma)\,c_2(\sigma_2^1(a_1^1, q)) \\ & + \gamma - c_1(a_1^1). \end{aligned}$$

The FOC's for maximization of π_1^1 and the corresponding payoff function for agent 2 yield the equilibrium effort in stage 1:

$$\sigma_1^i = (c_1')^{-1}((\alpha+\beta)\,\phi_2(0)) \quad \text{for } i = 1, 2.$$

We note in passing that exactly the same effort level is achieved in each stage under the "reverse" private feedback policy that informs each agent only of their *opponent's* performance instead of their own. The following proposition summarizes the above analysis.

Proposition 5.1. *Let f be any of the no-feedback, full-feedback, and private feedback policies. Then there exists a PBE σ under each f. The PBE outcome is unique, pure, and symmetric: The effort choice in stage 1 equals*

$$\sigma_1^i = (c_1')^{-1}((\alpha+\beta)\,\phi_2(0))$$

under any feedback policy. On the other hand, the equilibrium effort choice in stage 2 equals

$$\sigma_2^i = (c_2')^{-1}((\alpha+\beta)\,\phi_2(0)).$$

under the no-feedback policy,

$$\sigma_{2,0}^i(z_1) = \begin{cases} (c_2')^{-1}(\phi_2(0)) & \textit{if } z_1^1 = z_1^2, \\ 0 & \textit{otherwise.} \end{cases}$$

under the full-feedback policy, and

$$\sigma_{2,0}^i(\varepsilon_1^i) = \begin{cases} (c_2')^{-1}(\frac{\alpha}{\alpha+\gamma}\phi_2(0)) & \textit{if } \varepsilon_1^i = 0, \\ (c_2')^{-1}(\frac{\beta}{\beta+\gamma}\phi_2(0)) & \textit{if } \varepsilon_1^i = q. \end{cases}$$

under the private feedback policy.

Proof. See the Appendix. ■

The fact that the private feedback policy induces the same stage 1 effort as the other two is a consequence of our simplifying assumption (18) on noise. The above proposition allows us to compare the principal's payoffs under the three policies. Since the stage 1 effort is the same for every policy, the difference in the principal's payoffs arises solely from the stage 2 effort. The expected stage 2 effort can be computed as

$$a_2^N = (c_2')^{-1}((\alpha+\beta)\,\phi_2(0)).$$

under the no-feedback policy,

$$a_2^F = (\alpha+\beta)\,(c_2')^{-1}(\phi_2(0))$$

under the full-feedback policy, and

$$a_2^P = (\alpha+\gamma)\,(c_2')^{-1}\Big(\frac{\alpha}{\alpha+\gamma}\phi_2(0)\Big) + (\beta+\gamma)\,(c_2')^{-1}\Big(\frac{\beta}{\beta+\gamma}\phi_2(0)\Big)$$

under the private feedback policy. Suppose now that $(c_2')^{-1}$ is concave, then Jansen's inequality implies that

$$\begin{aligned} a_2^N &= (c_2')^{-1}\Big(((\alpha+\gamma)\,\frac{\alpha}{\alpha+\gamma}\,\phi_2(0) + (\beta+\gamma)\,\frac{\beta}{\beta+\gamma}\,\phi_2(0)\Big) \\ &\geq (\alpha+\gamma)\,(c_2')^{-1}\Big(\frac{\alpha}{\alpha+\gamma}\,\phi_2(0)\Big) + (\beta+\gamma)\,(c_2')^{-1}\Big(\frac{\beta}{\beta+\gamma}\,\phi_2(0)\Big) = a_2^P \\ &\geq \alpha(c_2')^{-1}(\phi_2(0)) + \beta(c_2')^{-1}(\phi_2(0)) \\ &= a_2^F. \end{aligned}$$

It follows that in this case, the no-feedback policy induces the highest expected effort followed by the private policy, and the full-feedback policy in this order. When $(c_2')^{-1}$ is convex, all the inequalities are reversed so that the ordering is exactly reversed as well. We summarize this observation in the following proposition.

Proposition 5.2. *An agent's stage 2 expected effort induced by each feedback policy in the PBE is ranked as follows:*

No-feedback $\geq$ *Private feedback* $\geq$ *Full-feedback*

if c_2' *is convex, and*

Full-feedback $\geq$ *Private feedback* $\geq$ *No-feedback*

if c_2' *is concave.*

Again, when c_2' is linear, the three are all identical. The above relations on the expected effort translate into the principal's expected payoff as follows. When the two agents' efforts are perfect substitutes $V(a) = \sum_{t=1}^{2} (a_t^1 + a_t^2)$, the principal's payoffs under the three policies are ordered in exactly the same way as above. On the other hand, when the two agents' efforts are complementary, then the principal's payoffs may have a different ordering: While the relative ranking of the full-feedback and no-feedback policies remains the same, the private feedback policy becomes less desirable given that it induces an asymmetric effort profile in stage 2 with probability 2γ. For example, if the two agents' efforts are perfect complements $V(a) = \sum_{t=1}^{2} \min \{a_t^1, a_t^2\}$, then it can be verified that the expected payoff under the private feedback policy is lower than that under the full-feedback policy for any c_2'. With perfect complementarity, the private policy induces a lower payoff than the no-feedback policy as well when c_2' is linear.

6 Mixed Equilibrium

In a dynamic contest such as the one studied in this paper, it is natural to think that an agent may attempt to manipulate his opponent by making some unexpected move early on. This is one expression of the strategic effect discussed in the Introduction. While no surprise spurt is possible in equilibrium, such an intuition may in part captured by a mixed equilibrium in which agents' stage 1 efforts are stochastic and their realizations become known to the opponent only ex post. In this section, we point out that a mixed choice of effort in stage 1 leads to a lower expected effort in stage 2.

The analysis of a mixed PBE is difficult for the following reasons. First, an agent's stage 2 effort is contingent on the realization of his stage 1 effort. This implies that the stage 2 effort profile is in general asymmetric between the agents as is the case with private feedback in the previous section. Second, it is difficult to identify sufficient conditions for the existence of an equilibrium in which an agent is indifferent between two or more effort choices. For these reasons, we will adopt the simplified framework of Section 5 under the additional simplification that the stage 1 effort choice is binary. Furthermore, we focus on the full-feedback policy since our interest is in an agent's incentive for strategic manipulation through the revealed information.

Formally, suppose that agent i's effort in stage 1 can be either low $a_1^i = 0$ or high $a_1^i = 1$. The cost of effort equals $c_1(0)$ when $a_1^i = 0$ and $c_1(1)$ when $a_1^i = 1$ with $0 \leq c_1(0) < c_1(1) < 1$. The stage 2 action can take any non-negative real value. The stage 1 noise has the same distribution as in Section 5, and the performance gap of q is unrecoverable in the sense of (18). Let σ be any pure or mixed PBE under the full-feedback policy. It can be verified that agent i's stage 2 effort given the announcement $z_1 = (z_1^1, z_1^2)$ equals ($i = 1, 2$):

$$\sigma_2^i(a_1^i, z_1) = \lambda(z_1^1 - z_1^2) \equiv (c_2')^{-1}(\phi_2(z_1^1 - z_1^2)) \text{ for any } z_1 \text{ and } a_1^i. \tag{20}$$

The stage 2 effort choice is hence pure in any PBE. Let then p_i be the probability with which agent i chooses low effort $a_1^i = 0$ in stage 1. When agent 2 plays according to σ^2, agent 1's overall payoff as a function of his stage 1 effort a_1^1 can be written as:

$$\pi_1^1(a_1^1) = (\alpha+\beta)\left[p_2\{\Phi_2(a_1^1) - c_2(\lambda(a_1^1))\} + (1-p_2)\{\Phi_2(a_1^1 - 1) - c_2(\lambda(a_1^1 - 1))\}\right] + \gamma - c_1(a_1^1).$$

Since $0 < p_1 < 1$ implies the indifference $\pi_1^1(1) = \pi_1^1(0)$, p_2 in a completely mixed equilibrium should satisfy

$$(\alpha+\beta)\Big[\Phi_2(1) - \Phi_2(0) + (2p_2 - 1)\{c_2(\lambda(0)) - c_2(\lambda(1))\}\Big] = c_1(1) - c_1(0).$$

Solving this for p_2, we readily obtain the following proposition.

Proposition 6.1. *Suppose $\phi_2(0) > \phi_2(1)$. Under the full feedback policy, there exists a PBE in which the stage 1 effort choice is completely mixed if and only if*

$$K = \frac{\frac{c_1(1)-c_1(0)}{\alpha+\beta} - \Phi_2(1) + \Phi_2(0)}{c_2(\lambda(0)) - c_2(\lambda(1))} \in (-1, 1), \tag{21}$$

where λ is as defined in (20). In the mixed PBE, the stage 1 effort is $a_1^i = 0$ with probability $p_i = (1+K)/2$, and $a_1^i = 1$ with probability $1 - p_i = (1-K)/2$. The stage 2 effort, on the other hand, equals $\lambda(0)$ when $a_1^1 = a_1^2$, and $\lambda(1)$ when $a_1^1 \neq a_1^2$. When (21) holds, there also exist two pure PBE in which stage 1 effort profiles are $(a_1^1, a_1^2) = (0, 0)$ and $(1, 1)$. The stage 2 effort in either pure PBE equals $\lambda(0)$.

It is important to note that in the mixed equilibrium, an agent's incentive to take a costly action in stage 1 comes from two sources. First, the higher effort increases the probability of winning. Second, the higher effort in stage 1 reduces the stage 2 cost: When only one agent has chosen a costly action in stage 1, both agents choose lower effort in stage 2 than when both have chosen low effort in stage 1. In fact, since $\phi_2(1) < \phi_2(0)$, the stage 2 effort $\lambda(1)$ after the stage 1 profile $a_1 = (1, 0)$ is lower than the stage 2 effort $\lambda(0)$ after $a_1 = (0, 0)$. While the first effect is present in the pure equilibrium as well, the second effect is unique to the mixed equilibrium. Since a mixed choice of effort entails a mismatch of stage 1 actions with positive probability, the stage 2 effort is lower on average in the mixed equilibrium than in the pure equilibria. When $\phi_2(1) > \phi_2(0)$, on the other hand, there exist one mixed PBE and two pure PBE under (21). The pure PBE are both asymmetric with the stage 1 profiles given by $(1, 0)$ and $(0, 1)$. It can be seen that in this case too, the stage 2 effort in the mixed PBE is lower on average than in the pure PBE. This observation is summarized below.

Corollary 6.2. *Under the conditions of Proposition 6.1, an agent's expected stage 2 effort in the mixed PBE is lower than that in either pure PBE.*

It immediately follows from the above that the mixed PBE is dominated by the pure PBE in which the stage 1 effort profile is $a_1 = (1, 1)$. We have no general comparison of the mixed PBE with the other pure PBE with $a_1 = (0, 0)$ in terms of the expected effort over the two stages. While obtained under a rather special set of assumptions, the above logic underlying the mixed equilibrium appears quite general and captures the essence of strategic manipulation: An agent would mix his action in stage 1 precisely because it leads to a lower effort in stage 2. When the possibility of such strategic manipulation is a major concern, hence, the full-feedback policy becomes less desirable in comparison with the no-feedback policy.[21]

7 Conclusion

The paper gives a first attempt to understand the use of the designer's private information in a dynamic tournament, and its conclusion shows that the optimal feedback depends sensitively on the functional form of the agents' disutility of effort. Although the present model abstracts from many important features of real tournaments, we believe that such sensitivity is at the core of the information revelation problem.

[21]Under some conditions, the no-feedback policy also admits a mixed PBE. However, if we take the view that the mixed action results from the strategic manipulation, then the mixed PBE is not plausible under the no-feedback policy.

We note that the analysis can be generalized to a T-stage tournament in which information feedback has many more dimensions than in the two-stage model. In particular, a feedback policy in a T-stage model is a contingent plan which determines not only the degree of information revelation but also its timing. For example, the principal may choose to reveal the stage 1 score before stage 3 if the stage 2 score is in some range, but withhold it until stage 4 otherwise. Under a slightly stronger set of assumptions on noise, we can prove that a symmetric PBE exists in the T-stage model when a feedback policy is even. Furthermore, among the class of even feedback policies, the no-feedback policy is optimal when the stage t marginal costs function are convex for $t = 2, \ldots, T$, and the feedback policy that reveals the absolute value of the stage score after every period, and hence is "most revealing" in the class of even policies, is optimal when the stage t marginal cost functions are concave.[22]

One key assumption of the present model is that the principal commits to the announced feedback policy for any realization of his private signal. It should be noted that such commitment is a standard assumption of mechanism design.[23] If the principal lacks commitment and optimizes his announcement after seeing his signal, then the problem becomes one of cheap-talk: He would choose an announcement that maximizes the stage 2 effort regardless of his private information. This in turn implies that the principal's announcement loses its informational content.[24] In other words, the assumption of no commitment is equivalent to no-feedback in the commitment framework. One way to interpret the principal's commitment to his feedback policy is by assuming the enforcement by a third party, who monitors the principal for a deviation from the announced policy. Under such an interpretation, however, it should be noted that not all policies are equally credible. For example, if the principal declares the use of the no-feedback policy, any release of information afterward is a clear deviation. On the other hand, if the principal announces the use of the full-feedback policy, his deviation cannot be detected unless his private information is verified. This suggests that the full-feedback policy has more credibility problems. Such a variation in credibility levels would be an important consideration in the enforcement interpretation. Alternatively, even when the private information

[22]See Aoyagi (2005).

[23]For example, it is common in the analysis of auctions to assume that an auctioneer retains his good if no bid reaches the reserve price.

[24]Kaplan and Zamir (2000) find that the auctioneer cannot exploit his private information on the bidders' valuation if he cannot commit to an announcement policy.

is not verifiable, it may still be possible to use statistical testing to enforce a feedback policy when the tournaments are repeated over time under the same policy. For example, a statistical test would reject the honesty of a tournament organizer who always reports a close race for the sake of spurring competition. The analysis of such a model, however, is not straightforward.

A Appendix

Recall that the set $f^{-1}(y)$ of stage 1 scores x_1 compatible with the announcement y either has positive measure or is countable by the regularity assumption. In the Appendix, we assume for simplicity that the set $f^{-1}(y)$ has positive measure. When it is countable, any integral over $f^{-1}(y)$ should be replaced by the corresponding summation.

Given a strategy profile σ and announcement y, we denote by $g_1^{\sigma,f}(x_1 \mid a_1^i, y)$ the conditional density of the stage 1 score x_1 when the stage 1 effort profile is (a_1^i, σ_1^j) (*i.e.,* when agent i chooses a possibly off-equilibrium action a_1^i while agent j chooses the equilibrium effort level). For $x_1 \in f^{-1}(y)$, the conditional density can be explicitly written as

$$g_1^{\sigma,f}(x_1 \mid a_1^1, y) = \frac{\phi_1(x_1 - a_1^1 + \sigma_1^2)}{\int_{f^{-1}(y)} \phi_1(x_1' - a_1^1 + \sigma_1^2)\, dx_1'}, \quad \text{and}$$
$$g_1^{\sigma,f}(x_1 \mid a_1^2, y) = \frac{\phi_1(x_1 - \sigma_1^1 + a_1^2)}{\int_{f^{-1}(y)} \phi_1(x_1' - \sigma_1^1 + a_1^2)\, dx_1'}.$$

Note in particular that $g_1^{\sigma,f}(\cdot \mid a_1^i, y)$ depends on σ only through the stage 1 profile σ_1. With slight abuse of notation, we define $g_1^{\sigma,f}(x_1 \mid y) = g_1^{\sigma,f}(x_1 \mid \sigma_1^i, y)$: the density of x_1 conditional on y when both agents choose their effort according to σ. Its explicit form is given by

$$g_1^{\sigma,f}(x_1 \mid y) = \frac{\phi_1(x_1 - \sigma_1^1 + \sigma_1^2)}{\int_{f^{-1}(y)} \phi_1(x_1' - \sigma_1^1 + \sigma_1^2)\, dx_1'}.$$

Proof of Theorem 3.1 Fix any PBE σ. Recall that $\pi_2^i(a_2^i \mid \sigma, a_1^i, y)$ represents agent i's expected payoff in stage 2 when he chooses a_2^i in stage 2, his history after stage 1 is $h_1^i = (a_1^i, y)$, and agent j plays according to the equilibrium strategy σ^j. For simplicity, write $\pi_2^i(a_2^i \mid a_1^i, y)$ for $\pi_2^i(a_2^i \mid \sigma, a_1^i, y)$. As seen in the text, we have

$$\frac{\partial \pi_2^1}{\partial a_2^1}(a_2^1 \mid a_1^1, y) = E^{\sigma,f}[\phi_2(a_2^1 - \sigma_{2,0}^2(y) + \tilde{x}_1) \mid a_1^1, y] - c_2'(a_2^1).$$

Since $\frac{\partial \pi_2^1}{\partial a_2^1}(0 \mid a_1^1, y) > 0$ by $c_2'(0) = 0$, the sequentially rational choice of effort in stage 2 $\sigma_2^1(a_1^1, y)$ (if any) must satisfy the FOC

$$c_2'\left(\sigma_2^1(a_1^1, y)\right) = E^{\sigma,f}[\phi_2(\sigma_2^1(a_1^1, y) - \sigma_{2,0}^2(y) + \tilde{x}_1) \mid a_1^1, y] \tag{22}$$

for every a_1^1. Since $\inf_{a \in \mathbf{R}_+} c_2''(a) > \sup_{x \in \mathbf{R}} \phi_2'(x)$ by assumption, we also have $\frac{\partial^2 \pi_2^1}{\partial (a_2^1)^2}(a_2^1 \mid a_1^1, y) < 0$. It then follows that the above FOC is indeed sufficient for global maximization, and also that $\sigma_2^1(a_1^1, y)$ is differentiable as a function of a_1^1 by the implicit function theorem. Likewise, agent 2's stage 2 action satisfies

$$c_2'\left(\sigma_2^2(a_1^2, y)\right) = E^{\sigma,f}[\phi_2(-\sigma_{2,0}^1(y) + \sigma_2^2(a_1^2, y) - \tilde{x}_1) \mid a_1^2, y] \tag{23}$$

for every a_1^2. On the equilibrium path where $a_1^i = \sigma_1^i$, we have $\sigma_2^i(\sigma_1^i, y) = \sigma_{2,0}^i(y)$ and $E^{\sigma,f}[\cdot \mid \sigma_1^i, y] = E^{\sigma,f}[\cdot \mid y]$. Hence, (22) and (23) show that $\sigma_{2,0}^1(y)$ and $\sigma_{2,0}^2(y)$ must satisfy

$$\sigma_{2,0}^1(y) = \sigma_{2,0}^2(y) = \alpha_2(y) \equiv (c_2')^{-1}\left(E^{\sigma,f}\left[\phi_2(\tilde{x}_1) \mid y\right]\right). \tag{24}$$

Now let $\pi_1^i(a_1^i) = \pi_1^i(a_1^i \mid \sigma)$ be agent i's (overall) expected payoff when he takes a_1^i in stage 1 and $\sigma_2^i(a_1^i, y)$ in stage 2, while agent j plays according to his equilibrium strategy σ^j. For $i = 1$, we have

$$\begin{aligned} &\pi_1^1(a_1^1) \\ &= -c_1(a_1^1) \\ &+ \int_{\mathbf{R}} \left\{\Phi_2\left(\sigma_2^1(a_1^1, f(x_1)) - \sigma_{2,0}^2(f(x_1)) + x_1\right) - c_2\left(\sigma_2^1(a_1^1, f(x_1))\right)\right\} \\ &\qquad\qquad \times \phi_1(x_1 - a_1^1 + \sigma_1^2)\, dx_1. \end{aligned} \tag{25}$$

Given that σ_2^1 is differentiable in a_1^1 as noted above, we use the envelope theorem to differentiate π_1^1:

$$\begin{aligned} &(\pi_1^1)'(a_1^1) \\ &= -\int_{\mathbf{R}} \Phi_2\left(\sigma_2^1(a_1^1, f(x_1)) - \sigma_{2,0}^2(f(x_1)) + x_1\right)\, \phi_1'(x_1 - a_1^1 + \sigma_1^2)\, dx_1 \\ &+ \int_{\mathbf{R}} c_2\left(\sigma_2^1(a_1^1, f(x_1))\right)\, \phi_1'(x_1 - a_1^1 + \sigma_1^2)\, dx_1 - c_1'(a_1^1). \end{aligned} \tag{26}$$

If the equilibrium stage 1 action $a_1^1 = \sigma_1^1$ is strictly positive, the FOC $(\pi_1^1)'(\sigma_1^1) = 0$ must hold. Since $\sigma_{2,0}^1(y) = \sigma_{2,0}^2(y)$ for any $y \in Y$ by (24), this FOC is equivalent to

$$\begin{aligned} c_1'(\sigma_1^1) = &-\int_{\mathbf{R}} \Phi_2(x_1)\, \phi_1'(x_1 - \sigma_1^1 + \sigma_1^2)\, dx_1 \\ &+ \int_{\mathbf{R}} c_2\left(\alpha_2(\sigma_1, f(x_1))\right)\, \phi_1'(x_1 - \sigma_1^1 + \sigma_1^2)\, dx_1. \end{aligned}$$

Changing variables of the first integral, and then integrating it by parts, we see that this is equivalent to the first line of (7). The symmetric argument shows that the second line of (7) is equivalent to the FOC for agent 2. ■

Proof of Theorem 3.2 Write $\varepsilon = \kappa/2$, where κ is as defined in (9). Suppose that $\sigma_1 = (\sigma_1^1, \sigma_1^2)$ solves (7). We construct a PBE as follows. First, for each a_1^1, $a_1^2 \in \mathbf{R}$, and $y \in Y$, let

$$\varphi_2^1(a_2^1 \mid a_1^1, y) = E^{\sigma,f}[\phi_2(a_2^1 - \alpha_2(\sigma_1, y) + \tilde{x}_1) \mid a_1^1, y] - c_2'(a_2^1),$$

and

$$\varphi_2^2(a_2^2 \mid a_1^2, y) = E^{\sigma,f}[\phi_2(\alpha_2(\sigma_1, y) - a_2^2 + \tilde{x}_1) \mid a_1^2, y] - c_2'(a_2^2).$$

Define $\sigma_2^1(a_1^1, y) > 0$ and $\sigma_2^2(a_1^2, y) > 0$ to be the unique solutions to

$$\varphi_2^1(a_2^1 \mid a_1^1, y) = 0 \quad \text{and} \quad \varphi_2^2(a_2^2 \mid a_1^2, y) = 0,$$

respectively. To see that such a solution exists, note that $\varphi_2^1(0 \mid a_1^1, y) > 0$ since $c_2'(0) = 0$ and $\phi_2 > 0$, and that $\varphi_2^1(a_2^1 \mid a_1^1, y) < 0$ for a_2^1 large enough since $\lim_{a\to\infty} c_2'(a) > \varepsilon > \sup_{x\in\mathbf{R}} \phi_2(x)$. Furthermore, it follows from $\inf_{a\in\mathbf{R}} c_2''(a) > \varepsilon > \sup_{x\in\mathbf{R}} \phi_2'(x)$ that

$$\frac{\partial \varphi_2^1}{\partial a_2^1}(a_2^1 \mid a_1^1, y) = -c_2''(a_2^1) + E^{\sigma,f}[\phi_2'(a_2^1 - \sigma_{2,0}^2(y) + x_1) \mid a_1^1, y] \tag{27}$$
$$< -\kappa + \varepsilon < 0.$$

Hence there indeed exists a unique solution $\sigma_2^1(a_1^1, y) > 0$ to $\varphi_2^1(a_2^1 \mid a_1^1, y) = 0$. The symmetric argument applies to agent 2. Note now that when $a_1^i = \sigma_1^i$, $a_2^i = \alpha_2(\sigma_1, y)$ solves $\varphi_2^i(a_2^i \mid \sigma_1^i, y) = 0$. We can hence replace $\alpha_2(\sigma_1, y)$ in the definition of $\varphi_2^1(a_2^1 \mid a_1^1, y)$ by $\sigma_{2,0}^2(y) = \sigma_2^2(\sigma_1^2, y)$, and see that $\varphi_2^1(a_2^1 \mid a_1^1, y) = 0$ is equivalent to the FOC $\frac{\partial \pi_2^1}{\partial a_2^1}(a_2^1 \mid a_1^1, y) = 0$ ((22) in the proof of Theorem 1) of agent 1's stage 2 payoff maximization problem. To see that $a_2^1 = \sigma_2^1(a_1^1, y)$ is a sequentially rational choice, it suffices to note that

$$\frac{\partial^2 \pi_2^1}{\partial (a_2^1)^2}(a_2^1 \mid a_1^1, y) = -c_2''(a_2^1) + E^{\sigma,f}[\phi_2'(a_2^1 - \sigma_{2,0}^2(y) + \tilde{x}_1) \mid a_1^1, y]$$
$$< -\kappa + \varepsilon < 0.$$

The same observation holds for agent 2.

We now turn to the analysis of stage 1 effort. As in the proof of Theorem 1, write $\pi_1^i(a_1^i) = \pi_1^i(a_1^i \mid \sigma)$ for agent i's overall payoff when he takes action a_1^i in stage 1 and chooses $\sigma_2^i(a_1^i, y)$ in stage 2, and agent j plays according to σ^2. Define

$$\begin{aligned}
\varphi_1^1(a_1^1) &= -c_1'(a_1^1) + \bar{\phi}(a_1^1 - \sigma_1^2) \\
&+ \int_{\mathbf{R}} c_2\left(\alpha_2(a_1^1, \sigma_1^2, f(x_1))\right)\, \phi_1'(x_1 - a_1^1 + \sigma_1^2)\, dx_1, \\
\varphi_1^2(a_1^2) &= -c_1'(a_1^2) + \bar{\phi}(\sigma_1^1 - a_1^2) \\
&- \int_{\mathbf{R}} c_2\left(\alpha_2(\sigma_1^1, a_1^2, f(x_1))\right)\, \phi_1'(x_1 - \sigma_1^1 + a_1^2)\, dx_1.
\end{aligned}$$

By assumption, $a_1^i = \sigma_1^i$ solves $\varphi_1^i(a_1^i) = 0$. Furthermore, $\varphi_1^i(a_1^i) = (\pi_1^i)'(a_1^i)$ as seen in the proof of Theorem 1 so that σ_1^i is a solution to the FOC of agent i's payoff maximization problem. In what follows, We will show $(\varphi_1^i)' = (\pi_1^i)'' < 0$ and hence σ_1^i is indeed the maximizer of π_1^1.

Since σ_2^1 is differentiable with respect to a_1^1 as noted in the proof of Theorem 3.1, we can differentiate (26) to obtain

$$\begin{aligned}
&(\varphi_1^1)'(a_1^1) = -c_1''(a_1^1) \\
&- \int_{\mathbf{R}} \left\{\phi_2\left(\sigma_2^1(a_1^1, f(x_1)) - \sigma_{2,0}^2(f(x_1)) + x_1\right) - c_2'(\sigma_2^1(a_1^1, f(x_1)))\right\} \\
&\qquad\qquad \times \frac{\partial \sigma_2^1}{\partial a_1^1}(a_1^1, f(x_1))\, \phi_1'(x_1 - a_1^1 + \sigma_1^2)\, dx_1 \\
&+ \int_{\mathbf{R}} \left\{\Phi_2\left(\sigma_2^1(a_1^1, f(x_1)) - \sigma_{2,0}^2(f(x_1)) + x_1\right) - c_2(\sigma_2^1(a_1^1, f(x_1)))\right\} \\
&\qquad\qquad \times \phi_1''(x_1 - a_1^1 + \sigma_1^2)\, dx_1.
\end{aligned}$$

Note now that for any $y \in Y$, we have $c_2'(\sigma_2^1(a_1^1, y)) \le \varepsilon$ by (23) and $c_2(\sigma_2^1(a_1^1, y)) \le 1$ by the above observation that $\sigma_2^1(a_1^1, y)$ maximizes $\pi_2^1(\cdot \mid a_1^1, y)$. Hence,

$$\left|\phi_2(\sigma_2^1(a_1^1, f(x_1)) - \sigma_{2,0}^2(f(x_1)) + x_1) - c_2'(\sigma_2^1(a_1^1, f(x_1)))\right| \le \varepsilon,$$

and

$$\left|\Phi_2(\sigma_2^1(a_1^1, f(x_1)) - \sigma_{2,0}^2(f(x_1)) + x_1) - c_2(\sigma_2^1(a_1^1, f(x_1)))\right| \le 1.$$

It follows that

$$\begin{aligned}
(\varphi_1^1)'(a_1^1) \le -c_1''(a_1^1) + \varepsilon \int_{\mathbf{R}} \left|\frac{\partial \sigma_2^1}{\partial a_1^1}(a_1^1, f(x_1))\right| \left|\phi_1'(x_1 - a_1^1 + \sigma_1^2)\right| dx_1 \\
+ \int_{\mathbf{R}} \left|\phi_1''(x_1 - a_1^1 + \sigma_1^2)\right| dx_1.
\end{aligned} \tag{28}$$

For $x_1 \in f^{-1}(y)$, we have

$$\frac{\partial g_1^{\sigma,f}}{\partial a_1^1}(x_1 \mid a_1^1, y) = \frac{-\phi_1'(x_1 - a_1^1 + \sigma_1^2)}{\int_{f^{-1}(y)} \phi_1(\hat{x}_1 - a_1^1 + \sigma_1^2)\, d\hat{x}_1} + \frac{\phi_1(x_1 - a_1^1 + \sigma_1^2) \int_{f^{-1}(y)} \phi_1'(\hat{x}_1 - a_1^1 + \sigma_1^2)\, d\hat{x}_1}{\left\{\int_{f^{-1}(y)} \phi_1(\hat{x}_1 - a_1^1 + \sigma_1^2)\, d\hat{x}_1\right\}^2},$$

and hence

$$\int_{\mathbf{R}} \left| \frac{\partial g_1^{\sigma,f}}{\partial a_1^1}(x_1 \mid a_1^1, y) \right| dx_1 \le 2 \frac{\int_{f^{-1}(y)} |\phi_1'(x_1 - a_1^1 + \sigma_1^2)|\, dx_1}{\int_{f^{-1}(y)} \phi_1(x_1 - a_1^1 + \sigma_1^2)\, dx_1}. \tag{29}$$

For simplicity, write $\hat{E}$ for $E^{\sigma,f}[\cdot \mid a_1^1]$, expectation with respect to x_1 given the stage 1 effort profile (a_1^1, σ_1^2). If we let

$$q(x_1) = \frac{\left|\phi_1'(x_1 - a_1^1 + \sigma_1^2)\right|}{\phi_1(x_1 - a_1^1 + \sigma_1^2)},$$

then (29) can be written as

$$\int_{\mathbf{R}} \left| \frac{\partial g_1^{\sigma,f}}{\partial a_1^1}(x_1 \mid a_1^1, y) \right| dx_1 \le 2\hat{E}[q(\tilde{x}_1) \mid y].$$

On the other hand,

$$\frac{\partial \varphi_2^1}{\partial a_1^1}(a_2^1 \mid a_1^1, y) = \int_{\mathbf{R}} \phi_2\left(a_2^1 - \sigma_{2,0}^2(y) + x_1\right) \frac{\partial g_1^{\sigma,f}}{\partial a_1^1}(x_1 \mid a_1^1, y)\, dx_1,$$

so that

$$\left| \frac{\partial \varphi_2^1}{\partial a_1^1}(a_2^1 \mid a_1^1, y) \right| < \varepsilon \int_{\mathbf{R}} \left| \frac{\partial g_1^{\sigma,f}}{\partial a_1^1}(x_1 \mid a_1^1, y) \right| dx_1 \le 2\varepsilon\, \hat{E}[q(\tilde{x}_1) \mid y].$$

Therefore,

$$\begin{aligned}
&\frac{1}{2\varepsilon} \int_{\mathbf{R}} \left| \frac{\partial \varphi_2^1}{\partial a_1^1}(a_2^1 \mid a_1^1, y = f(x_1)) \right| \left|\phi_1'(x_1 - a_1^1 + \sigma_1^2)\right| dx_1 \\
&\le \int_{\mathbf{R}} \hat{E}[q(\tilde{x}_1) \mid \tilde{y} = f(x_1)]\, q(x_1)\, \phi_1(x_1 - a_1^1 + \sigma_1^2)\, dx_1 \\
&= \hat{E}\left[\hat{E}[q(\tilde{x}_1) \mid \tilde{y}]\, q(\tilde{x}_1)\right] \\
&\le \hat{E}\left[\hat{E}[q(\tilde{x}_1) \mid \tilde{y}]^2\right]^{1/2} \hat{E}\left[q(\tilde{x}_1)^2\right]^{1/2} \\
&\le \hat{E}\left[\hat{E}[q(\tilde{x}_1)^2 \mid \tilde{y}]\right]^{1/2} \hat{E}\left[q(\tilde{x}_1)^2\right]^{1/2} \\
&= \hat{E}\left[q(\tilde{x}_1)^2\right] = \int_{\mathbf{R}} \left| \frac{\phi_1'(x_1)}{\phi_1(x_1)} \right|^2 \phi_1(x_1)\, dx_1 < \varepsilon,
\end{aligned}$$

where the fourth line follows from Schwartz' inequality and the fifth line from Jensen's inequality. Using the implicit function theorem and evaluating $\frac{\partial \varphi_2^1}{\partial a_2^1}$ using (27), we see that the second term on the right-hand side of (28) satisfies

$$\begin{aligned}
&\varepsilon \int_{\mathbf{R}} \left| \frac{\partial \sigma_2^1}{\partial a_1^1}(a_1^1, f(x_1)) \right| \left| \phi_1'(x_1 - a_1^1 + \sigma_1^2) \right| dx_1 \\
&= \varepsilon \int_{\mathbf{R}} \frac{\left| \frac{\partial \varphi_2^1}{\partial a_1^1}(\sigma_2^1(a_1^1, f(x_1)) \mid a_1^1, y = f(x_1)) \right|}{\left| \frac{\partial \varphi_2^1}{\partial a_2^1}(\sigma_2^1(a_1^1, f(x_1)) \mid a_1^1, y = f(x_1)) \right|} \left| \phi_1'(x_1 - a_1^1 + \sigma_1^2) \right| dx_1 \\
&\leq \frac{\varepsilon}{\kappa - \varepsilon} \int_{\mathbf{R}} \left| \frac{\partial \varphi_2^1}{\partial a_1^1}(\sigma_2^1(a_1^1, f(x_1)) \mid a_1^1, f(x_1)) \right| \left| \phi_1'(x_1 - a_1^1 + \sigma_1^2) \right| dx_1 \\
&\leq \frac{2\varepsilon^3}{\kappa - \varepsilon}
\end{aligned}$$

Hence,

$$(\varphi_1^1)'(a_1^1) \leq -\kappa + \frac{2\varepsilon^3}{\kappa - \varepsilon} + \varepsilon < 0.$$

This proves the claim. ■

Proof of Theorem 3.3 Suppose that $\sigma_1^1 = \sigma_1^2$. We first show that $\alpha_2(\sigma_1, f(x_1)) = \alpha_2(\sigma_1, f(-x_1))$ for any x_1. This would hold trivially if f is even since then $f(x_1) = f(-x_1)$. If f is odd, then $g_1^{\sigma,f}(x_1 \mid y) = g_1^{\sigma,f}(-x_1 \mid -y)$, and hence the symmetry of ϕ_2 implies that

$$\begin{aligned}
\alpha_2(\sigma_1, y) &= (c_2')^{-1} \left(\int_{\mathbf{R}} \phi_2(x_1)\, g_1^{\sigma,f}(x_1 \mid y)\, dx_1 \right) \\
&= (c_2')^{-1} \left(\int_{\mathbf{R}} \phi_2(-x_1)\, g_1^{\sigma,f}(-x_1 \mid -y)\, dx_1 \right) \\
&= \alpha_2(\sigma_1, -y).
\end{aligned}$$

It follows that $\alpha_2(\sigma_1, f(-x_1)) = \alpha_2(\sigma_1, -f(x_1)) = \alpha_2(\sigma_1, f(x_1))$. With this equality, $\sigma_1^1 = \sigma_1^2 = a_1^*$ solves (7) since

$$\begin{aligned}
&\int_{\mathbf{R}} c_2(\alpha_2(\sigma_1, f(x_1)))\, \phi_1'(x_1)\, dx_1 \\
&= \int_0^{\infty} c_2(\alpha_2(\sigma_1, f(x_1)))\, \phi_1'(x_1)\, dx_1 + \int_{-\infty}^{0} c_2(\alpha_2(\sigma_1, f(x_1)))\, \phi_1'(x_1)\, dx_1 \\
&= \int_0^{\infty} c_2(\alpha_2(\sigma_1, f(x_1)))\, \phi_1'(x_1)\, dx_1 - \int_0^{\infty} c_2(\alpha_2(\sigma_1, f(-x_1)))\, \phi_1'(x_1)\, dx_1 \\
&= 0.
\end{aligned}$$

This completes the proof. ■

Lemma A.1. *Suppose that Assumptions 1 and 2 hold and that* $\lim_{a\to\infty} c_1'(a) > 2\bar{\phi}(0)$ *for* $i = 1, 2$*. Then for any* σ_1 *that solves (7) and any* a_2 *such that* $a_2^1 = a_2^2$*, the principal's payoff function satisfies*

$$V((a_1^*, a_1^*), a_2) \geq V(\sigma_1, a_2).$$

Proof. Fix $a_2 \in \mathbf{R}_+^2$ such that $a_2^1 = a_2^2$. Since h is continuous, the inverse image $h^{-1}(\{0\})$ is closed. Furthermore, it is non-empty since $(a_1^*, a_1^*) \in h^{-1}(\{0\})$. To see that it is bounded, note that $(c_1')^{-1}(2\bar{\phi}(0)) < \infty$ by assumption. For any a_1 such that $\max\{a_1^1, a_1^2\} > (c_1')^{-1}(2\bar{\phi}(0))$, we have

$$h(a_1) \geq c_1'(a_1^1) + c_1'(a_1^2) - 2\bar{\phi}(0) > 0,$$

where the first inequality follows from Assumption 2 and the second from the monotonicity of c_1'. This shows that $h^{-1}(\{0\})$ is a subset of the bounded set $\{a_1 : \max\{a_1^1, a_1^2\} \leq (c_1')^{-1}(2\bar{\phi}(0))\}$ and hence is compact. It follows that the continuous function $V(\cdot, a_2)$ on the compact set $h^{-1}(\{0\}) = \{a_1 \in \mathbf{R}_+^2 : h(a_1) = 0\}$ achieves a maximum. Let $\bar{a}_1 = (\bar{a}_1^1, \bar{a}_1^2) \in h^{-1}(\{0\})$ be any maximizer of $V(\cdot, a_2)$ in $h^{-1}(\{0\})$. We show that $\bar{a}_1 = (a_1^*, a_1^*)$. Suppose that $\bar{a}_1^1 < \bar{a}_1^2$. Since $\frac{\partial h}{\partial a_1^2} \neq 0$ by (13), the implicit function theorem shows that there exists a function γ defined in a neighborhood of $\bar{a}_1^1$ such that $h(a_1^1, \gamma(a_1^1)) = 0$. Furthermore, γ is differentiable at $\bar{a}_1^1$ and the derivative $\gamma'(\bar{a}_1^1)$ is given by the left-hand side of (13) with $\bar{a}_1^i$ replacing a_1^i. Now let $\delta(a_1^1) = V((a_1^1, \gamma(a_1^1)), a_2)$. δ is also differentiable at $\bar{a}_1^1$ and its derivative is given by

$$\delta'(\bar{a}_1^1) = \frac{\partial V}{\partial a_1^1}(\bar{a}_1, a_2) + \frac{\partial V}{\partial a_1^2}(\bar{a}_1, a_2)\, \gamma'(\bar{a}_1^1).$$

It can be readily verified that Assumption 1 implies $\delta'(\bar{a}_1^1) > 0$. This contradicts our assumption that V is maximized at $\bar{a}_1$ in $h^{-1}(\{0\}) = 0$. The symmetric argument shows that it cannot be maximized at $\bar{a}$ such that $\bar{a}_1^1 > \bar{a}_1^2$ either. Hence, we must have $\bar{a}_1^1 = \bar{a}_1^2 = a_1^*$. ■

Proof of Theorem 4.4 Let f be any feedback policy that admits a PBE σ for which (7) holds. As in the proof of Theorem 4.1, Jensen's inequality and the law of iterated expectation applied to (11) imply that the expected stage 2 effort satisfies

$$
\begin{aligned}
E^{\sigma,f}[\alpha_2(\sigma_1,\tilde{y})] &= E^{\sigma,f}\left[(c_2')^{-1}\left(E^{\sigma,f}[\phi_2(\tilde{x}_1)\mid \tilde{y}]\right)\right]\\
&\leq (c_2')^{-1}\left(E^{\sigma,f}\left[E^{\sigma,f}[\phi_2(\tilde{x}_1)\mid \tilde{y}]\right]\right)\\
&= (c_2')^{-1}\left(E^{\sigma,f}[\phi_2(\tilde{x}_1)]\right)\\
&= (c_2')^{-1}\left(\bar{\phi}(\sigma_1^1-\sigma_1^2)\right)\\
&\leq (c_2')^{-1}\left(\bar{\phi}(0)\right) = a_2^N,
\end{aligned}
$$

where the last inequality follows from Assumption 2. It hence follows from (3) that

$$v(\sigma,f) = E^{\sigma,f}\left[V\left(\sigma_1, a_2^1 = a_2^2 = \alpha_2(\sigma_1,\tilde{y})\right)\right] \leq V(\sigma_1,(a_2^N,a_2^N)).$$

Since σ_1 solves (7) by assumption,

$$V\left(\sigma_1,(a_2^N,a_2^N)\right) \leq V\left((a_1^*,a_1^*),(a_2^N,a_2^N)\right)$$

by Lemma A.1. Since the right-hand side of the above inequality equals the principal's expected payoff in the symmetric PBE under the no-feedback policy, the desired conclusion follows. ■

Proof of Theorem 4.5 We first show that Assumption 3 implies

(30) $$P(|\tilde{\zeta}_2| \geq \kappa)) = \min_{\delta\in\mathbf{R}} P(|\tilde{\zeta}_2+\delta| \geq \kappa)) \text{ for any } \kappa > 0.$$

Let $\delta > 0$ and $\kappa > 0$ be given. When $\delta < 2\kappa$, we have

$$
\begin{aligned}
& P(|\tilde{\zeta}_2| < \kappa)) - P(|\tilde{\zeta}_2+\delta| < \kappa))\\
= & -\int_{-\kappa-\delta}^{-\kappa}\phi_2(x)\,dx + \int_{\kappa-\delta}^{\kappa}\phi_2(x)\,dx\\
> & -\delta\,\phi_2(-\kappa) + \delta\,\phi_2(\kappa)\\
= & \ 0.
\end{aligned}
$$

On the other hand, when $\delta > 2\kappa$, we have

$$
\begin{aligned}
& P(|\tilde{\zeta}_2| < \kappa)) - P(|\tilde{\zeta}_2+\delta| < \kappa))\\
= & \int_{-\kappa}^{\kappa}\phi_2(x)\,dx - \int_{-\kappa-\delta}^{\kappa-\delta}\phi_2(x)\,dx\\
> & \ 2\kappa\,\phi_2(\kappa) - 2\kappa\,\phi_2(\kappa-\delta)\\
> & \ 0.
\end{aligned}
$$

The similar argument proves (30) when $\delta < 0$.

We now show that the expected stage 2 effort implied by σ is less than or equal to that implied by the symmetric PBE under the full-feedback policy:

$$E^{\sigma,f}\left[\alpha_2(\sigma_1,\tilde{y})\right] \leq a_2^F \equiv \int_{\mathbf{R}} (c_2')^{-1}(\phi_2(x_1))\,\phi_1(x_1)\,dx_1. \tag{31}$$

By the same logic as in the proof of Theorem 4.4, it would then follow from Lemma A.1 that $v(\sigma, f)$ is $\leq$ the principal's expected payoff in the symmetric PBE under the full-feedback policy.

Note that since $E^{\sigma,f}\left[\alpha_2(\sigma_1,\tilde{y})\right] \leq E^{\sigma,f}\left[(c_2')^{-1}(\phi_2(\tilde{x}_1))\right]$ as in the proof of Theorem 4.2, (31) is implied by

$$E^{\sigma,f}\left[(c_2')^{-1}(\phi_2(\tilde{x}_1))\right] \leq a_2^F. \tag{32}$$

Let $\eta_2 : [0, \phi_2(0)] \to \mathbf{R}_+$ be the inverse of the restriction of ϕ_2 to $\mathbf{R}_+$ with $\eta_2(0) = \infty$. In other words, for each $u \in [0, \phi_2(0)]$, $\eta_2(u) \geq 0$ is the unique number such that $\phi_2(\eta_2(u)) = u$. Note that η_2 is well-defined under Assumption 3. Given any $\delta \in \mathbf{R}$, let the function $G(\cdot \mid \delta) : [0, \phi_2(0)] \to \mathbf{R}_+$ be defined by $G(u \mid \delta) = 1 - \Phi_2(\eta_2(u) - \delta) + \Phi_2(-\eta_2(u) - \delta) = P\left(|\tilde{\zeta}_2 + \delta| \geq \eta_2(u)\right)$. Note that $G(\cdot \mid \delta)$ is a distribution function over $[0, \phi_2(0)]$ since it is increasing, and satisfies $G(0 \mid \delta) = 0$ and $G(\phi_2(0) \mid \delta) = 1$. If we write $\delta = \sigma_1^1 - \sigma_1^2$, then

$$\begin{aligned}
& E\left[(c_2')^{-1}\left(\phi_2(x_1)\right) \mid a_1\right] \\
= & \int_{\mathbf{R}} (c_2')^{-1}\left(\phi_2(x_1)\right)\,\phi_1(x_1 - \delta)\,dx_1 \\
= & \int_0^{\phi_2(0)} (c_2')^{-1}(u)\,\phi_1(\eta_2(u) - \delta)\,(-\eta_2'(u))\,du \\
+ & \int_0^{\phi_2(0)} (c_2')^{-1}(u)\,\phi_1(-\eta_2(u) - \delta)\,(-\eta_2'(u))\,du \\
= & \int_0^{\phi_2(0)} (c_2')^{-1}(u)\,dG(u \mid \delta),
\end{aligned}$$

where the second equality follows from first dividing the range of the integral and then applying the change of variables from x_1 to $u = \phi_2(x_1)$ or $u = -\phi_2(x_1)$. By (30), $G(u \mid \delta) = P\left(|\tilde{\zeta}_2 + \delta| \geq \eta_2(u)\right) \geq P\left(|\tilde{\zeta}_2| \geq \eta_2(u)\right) = G(u \mid 0)$ for any $u \in [0, \phi_2(0)]$ and $\delta \in \mathbf{R}$ so that $G(u \mid 0)$ first-order stochastically dominates $G(u \mid \delta)$ with $\delta \neq 0$. Since $(c_2')^{-1}$ is increasing, it follows that

$$\int_0^{\phi_2(0)} (c_2')^{-1}(u)\,dG(u \mid \delta) \leq \int_0^{\phi_2(0)} (c_2')^{-1}(u)\,dG(u \mid 0).$$

Changing variables back to x_1, we see that the right-hand side of this inequality equals a_2^F. ■

Proof of Theorem 4.8 For any $\hat{r} \in (r, (c_2')^{-1}(\phi_2(0)))$, define

$$\begin{aligned} p &= \int_{\{x_1 : \phi_2(x_1) \geq c_2'(\hat{r})\}} \phi_1(x_1)\, dx_1, \\ s &= \frac{1}{1-p} \int_{\{x_1 : \phi_2(x_1) < c_2'(\hat{r})\}} \phi_2(x_1)\, \phi_1(x_1)\, dx_1, \\ t &= \frac{1}{p} \int_{\{x_1 : \phi_2(x_1) \geq c_2'(\hat{r})\}} \phi_2(x_1)\, \phi_1(x_1)\, dx_1. \end{aligned}$$

Now consider the even feedback policy f that only reveals whether $\phi_2(x_1) \geq c_2'(\hat{r})$ or not. We can express the expected stage 2 effort in the symmetric PBE σ under f as

$$\begin{aligned} & E^{\sigma,f}\left[(c_2')^{-1}(E^{\sigma,f}[\phi_2(\tilde{x}_1) \mid \tilde{y}])\right] \\ &= E^{\sigma,f}\left[(c_2')^{-1}(E^{\sigma,f}[\phi_2(\tilde{x}_1) \mid \tilde{y}]) \,\middle|\, \phi_2(\tilde{x}_1) \geq c_2'(\hat{r})\right] P^{\sigma,f}\left(\phi_2(\tilde{x}_1) \geq c_2'(\hat{r})\right) \\ &+ E^{\sigma,f}\left[(c_2')^{-1}(E^{\sigma,f}[\phi_2(\tilde{x}_1) \mid \tilde{y}]) \,\middle|\, \phi_2(\tilde{x}_1) < c_2'(\hat{r})\right] P^{\sigma,f}\left(\phi_2(\tilde{x}_1) < c_2'(\hat{r})\right) \\ &= (1-p)\,(c_2')^{-1}(s) + p(c_2')^{-1}(t), \end{aligned}$$

where the second equality follows since the functions inside the expectations are constant over the conditioning events by assumption. On the other hand, the stage 2 effort in the symmetric PBE under the no-feedback policy can be expressed as

$$(c_2')^{-1}((1-p)\,s + p\,t).$$

Since $s \to \bar{\phi}(0) > c_2'(r)$ as $\hat{r} \to (c_2')^{-1}(\phi_2(0))$, we can take $\hat{r}$ close enough to $(c_2')^{-1}(\phi_2(0))$ so that $s > c_2'(r)$. Since $t \geq s$, we have $s,\, t \in [c_2'(r), \phi_2(0)]$ for such an r. Given that $(c_2')^{-1}$ is convex over this interval by assumption, it follows that

$$(1-p)\,(c_2')^{-1}(s) + p(c_2')^{-1}(t) \geq (c_2')^{-1}((1-p)\,s + p\,t). \tag{33}$$

This proves the claim. ■

Proof of Proposition 5.1 Take first the private feedback policy. Differentiation of agent 1's stage 2 payoff function yields

$$\frac{\partial \pi_2^1}{\partial a_2^1}(a_2^1 \mid a_1^1, \varepsilon_1^1) = P(\tilde{\varepsilon}_1^2 = \varepsilon_1^1 \mid \varepsilon_1^1)\, \phi_2\Big(a_1^1 - \sigma_1^2 + a_2^1 - \sigma_{2,0}(\varepsilon_1^1)\Big) - c_2'(a_2^1).$$

From this follows the FOC in the text. Differentiating this, we also obtain the second-order derivative:

$$\frac{\partial^2 \pi_2^1}{\partial (a_2^1)^2}(a_2^1 \mid a_1^1, \varepsilon_1^1) = P(\tilde{\varepsilon}_1^2 = \varepsilon_1^1 \mid \varepsilon_1^1)\, \phi_2'\Big(a_1^1 - \sigma_1^2 + a_2^1 - \sigma_{2,0}(\varepsilon_1^1)\Big) - c_2''(a_2^1) \tag{34}$$
$$< 0,$$

where the inequality follows from (19). Hence, the FOC characterizes the optimal choice. For the equilibrium choice of stage 1 effort $a_1^1 = \sigma_1^1$, $a_2^1 = \sigma_2^1(\sigma_1^1, \varepsilon_1^1) = \sigma_{2,0}^1(\varepsilon_1^1)$ satisfies the following FOC:

$$P(\tilde{\varepsilon}_1^2 = \varepsilon_1^1 \mid \varepsilon_1^1)\, \phi_2\Big(\sigma_1^1 - \sigma_1^2 + \sigma_{2,0}^1(\varepsilon_1^1) - \sigma_{2,0}^2(\varepsilon_1^1)\Big) = c_2'(\sigma_{2,0}^1(\varepsilon_1^1)).$$

The corresponding condition for agent 2 is given by

$$P(\tilde{\varepsilon}_1^1 = \varepsilon_1^2 \mid \varepsilon_1^2)\, \phi_2\Big(\sigma_1^2 - \sigma_1^1 + \sigma_{2,0}^2(\varepsilon_1^2) - \sigma_{2,0}^1(\varepsilon_1^2)\Big) = c_2'(\sigma_{2,0}^2(\varepsilon_1^2)).$$

It follows from these FOC's that

$$\sigma_{2,0}^1(\varepsilon_1^i) = \sigma_{2,0}^2(\varepsilon_1^i) = (c_2')^{-1}\Big(P(\tilde{\varepsilon}_1^j = \varepsilon_1^i \mid \varepsilon_1^i)\, \phi_2(\sigma_1^2 - \sigma_1^1)\Big)$$

as given in the text. The inequality (34) also shows that agent 1's sequentially rational effort choice $\sigma_2^1(a_1^1, \varepsilon_1^1)$ in stage 2 is uniquely determined and is differentiable as a function of a_1^1. By the implicit function theorem, the derivative is given by

$$\frac{\partial \sigma_2^1}{\partial a_1^1}(a_1^1, \varepsilon_1^1) = \frac{P(\tilde{\varepsilon}_1^2 = \varepsilon_1^1 \mid \varepsilon_1^1)\, \phi_2'(a_1^1 - \sigma_1^2 + a_2^1 - \sigma_{2,0}(\varepsilon_1^1))}{c_2''(\sigma_2(a_1^1, \varepsilon_1^1)) - P(\tilde{\varepsilon}_1^2 = \varepsilon_1^1 \mid \varepsilon_1^1)\, \phi_2'(a_1^1 - \sigma_1^2 + a_2^1 - \sigma_{2,0}(\varepsilon_1^1))}.$$

It follows from (19) that

$$\Big|\frac{\partial \sigma_2^1}{\partial a_1^1}(a_1^1, \varepsilon_1^1)\Big| \leq \frac{\sup\ \phi_2'(x)}{\inf\ c_2''(a) - \sup\ \phi_2'(x)} \leq \frac{1}{\kappa - 1}. \tag{35}$$

As in the text, let $\pi_1^1(a_1^1)$ denote agent 1's overall payoff when his stage 1 effort equals a_1^1:

$$\begin{aligned}\pi_1^1(a_1^1) = {} & \alpha\, \Phi_2\Big(a_1^1 - \sigma_1^2 + \sigma_2^1(a_1^1, 0) - \sigma_{2,0}^2(0)\Big) - (\alpha + \gamma)\, c_2(\sigma_2^1(a_1^1, 0)) \\ & + \beta\, \Phi_2\Big(a_1^1 - \sigma_1^2 + \sigma_2^1(a_1^1, q) - \sigma_{2,0}^2(q)\Big) - (\beta + \gamma)\, c_2(\sigma_2^1(a_1^1, q)) \\ & + \gamma - c_1(a_1^1).\end{aligned}$$

Given that σ_2^1 is differentiable with respect to a_1^1, we can differentiate π_1 using the envelope theorem to get

$$\begin{aligned}(\pi_1^1)'(a_1^1) = {} & \alpha\, \phi_2\Big(a_1^1 - \sigma_1^2 + \sigma_2^1(a_1^1, 0) - \sigma_{2,0}^2(0)\Big) \\ & + \beta\, \phi_2\Big(a_1^1 - \sigma_1^2 + \sigma_2^1(a_1^1, q) - \sigma_{2,0}^2(q)\Big) - c_1'(a_1^1).\end{aligned}$$

The second-order derivative is given by

$$(\pi_1^1)''(a_1^1) = \alpha\,\phi_2'\Big(a_1^1 - \sigma_1^2 + \sigma_2^1(a_1^1, 0) - \sigma_{2,0}^2(0)\Big)\left\{1 + \frac{\partial \sigma_2^1}{\partial a_1^1}(a_1^1, 0)\right\}$$
$$+ \beta\,\phi_2'\Big(a_1^1 - \sigma_1^2 + \sigma_2^1(a_1^1, q) - \sigma_{2,0}^2(q)\Big)\left\{1 + \frac{\partial \sigma_2^1}{\partial a_1^1}(a_1^1, q)\right\} - c_1''(a_1^1).$$

Using (19) and (35), we can evaluate this as

$$(\pi_1^1)''(a_1^1) \leq (\alpha + \beta)\left(1 + \frac{1}{\kappa - 1}\right) \sup\, \phi_2'(x) - \inf\, c_1''(a) < 0.$$

It follows that the following FOC condition characterizes the optimal effort choice σ_1^1 in stage 1:

$$\alpha\,\phi_2(\sigma_1^1 - \sigma_1^2) + \beta\,\phi_2(\sigma_1^1 - \sigma_1^2) = c_1'(\sigma_1^1),$$

where we use the fact that $\sigma_{2,0}^1(\varepsilon_1) = \sigma_{2,0}^2(\varepsilon_1)$ for $\varepsilon_1 = 0,\, q$. Likewise, the FOC for agent 2's optimal stage 1 effort is given by

$$\alpha\,\phi_2(\sigma_1^1 - \sigma_1^2) + \beta\,\phi_2(\sigma_1^1 - \sigma_1^2) = c_1'(\sigma_1^2).$$

It follows that

$$\sigma_1^1 = \sigma_1^2 = (c_1')^{-1}((\alpha + \beta)\,\phi_2(0)).$$

Take next the full-feedback policy which reveals $z_1 = (z_1^1, z_1^2) = (a_1^1 + \varepsilon_1^1, a_1^2 + \varepsilon_1^2)$ at the end of stage 1. Since the stage 1 signal does not have full support unlike in the previous analysis, some realizations of z_1 are interpreted as off the equilibrium path. An agent's belief following any such z_1 is not uniquely determined. In other words, when agent 1, say, observes 2's performance level z_1^2 to be different from σ_1^2 or $\sigma_1^2 + q$, then Bayes rule does not pin down 1's belief about 2's stage 1 effort. In this sense, let $\mu_1(\cdot \mid z_1)$ be the probability measure over $\mathbf{R}_+$ representing agent 1's belief about agent 2's stage 1 effort when observing z_1. For any z_1, we can write agent 1's stage 2 payoff function as

$$\pi_2^1(a_2^1 \mid a_1^1, z_1)$$
$$= \int_{\mathbf{R}_+} \Phi_2\Big(z_1^1 - z_1^2 + a_2^1 - \sigma_2^2(a_1^2, z_1)\Big)\, d\mu_1(a_1^2 \mid z_1) - c_2(a_2^1).$$

However, when agent 2 plays according to the equilibrium, $z_1^2 = \sigma_1^2$ or $\sigma_1^2 + q$ and hence Bayes rule dictates that μ_1 place probability one on σ_1^2, *i.e.*,

$$\mu_1(\{\sigma_1^2\} \mid z_1) = 1 \quad \text{if } z_1^2 = \sigma_1^2 \text{ or } \sigma_1^2 + q.$$

Suppose from now on that z_1^2 takes one of these values. In this case, differentiation of the above payoff function yields

$$\frac{\partial \pi_2^1}{\partial a_2^1}(a_2^1 \mid a_1^1, z_1) = \phi_2\Big(z_1^1 - z_1^2 + a_2^1 - \sigma_{2,0}^2(z_1)\Big) - c_2'(a_2^1).$$

Since $c_2'(0) = 0$, $\sigma_2^1(a_1^1, z_1)$ satisfies the following FOC:

$$\phi_2\Big(z_1^1 - z_1^2 + \sigma_2^1(a_1^1, z_1) - \sigma_2^2(a_1^2, z_1)\Big) = c_2'(\sigma_2^1(a_1^1, z_1)).$$

The second-order derivative of π_2^1 is strictly negative by (19), and hence the above FOC characterizes the optimal effort choice. The equilibrium effort $\sigma_{2,0}^1(z_1)$ satisfies the above when $a_1^1 = \sigma_1^1$. This and the corresponding condition for agent 2 yield

$$\sigma_{2,0}^1(z_1) = \sigma_{2,0}^1(z_1) = (c_2')^{-1}(\phi_2(z_1^1 - z_1^2)).$$

The rest of the argument parallels that given above for the private feedback policy and is omitted. Finally, for the no-feedback policy, agent 1's stage 2 payoff function is given by

$$\pi_2^1(a_2^1 \mid a_1^1) = (\alpha + \beta)\, \Phi_2(a_1^1 - \sigma_1^2 + a_2^1 - \sigma_{2,0}^2) - c_2(a_2^1).$$

The FOC, which characterizes the optimal choice by (19), is given by

$$(\alpha + \beta)\, \phi_2(a_1^1 - \sigma_1^2 + \sigma_2^1(a_1^1) - \sigma_{2,0}^2) = c_2'(\sigma_2^1(a_1^1)).$$

The equilibrium effort choice $\sigma_{2,0}^1$ satisfies the above when $a_1^1 = \sigma_1^1$. This and the corresponding condition for agent 2 yield

$$\sigma_{2,0}^1 = \sigma_{2,0}^2 = (c_2')^{-1}((\alpha + \beta)\, \phi_2(\sigma_1^1 - \sigma_1^2)).$$

The rest of the argument is again similar and is omitted. ■

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