Journal of Economic Theory, 102, 229-248, 2002.

Collusion in Dynamic Bertrand Oligopoly with Correlated Private Signals and Communicationy

Masaki Aoyagiz

July 27, 1997

Revised: April 2, 2000

yI am grateful to David Cooper, Wolfgang Pesendorfer, and Phil Reny for helpful comments. Comments of an anonymous referee and an associate editor of this journal were particularly useful in the revision stage. Remaining errors are mine. This is a substantially revised version of Aoyagi [3].

zISER, Osaka University, and Department of Economics, University of Pittsburgh, Pittsburgh, PA 15260. E-mail address: maoyagi+@pitt.edu

Proposed running title: Collusion with Private Signals

Masaki Aoyagi
Department of Economics
University of Pittsburgh
Pittsburgh, PA 15260

Abstract

This paper studies collusion in repeated Bertrand oligopoly when stochastic demand levels for the product of each firm are their private information and are positively correlated. It derives general sufficient conditions for efficient collusion through communication and a simple grim-trigger strategy. This analysis is then applied to a model where the demand signal has multiple random components which respond differently to price deviations. In this model, it is shown that the above sufficient conditions hold if idiosyncratic noise terms are sufficiently small.

Keywords: private monitoring, correlation, repeated game, secret price cutting.
Journal of Economic Literature Classification Numbers: C72, D82.

1. Introduction

In a seminal paper, Stigler [24] studies collusion in repeated Bertrand oligopoly when firms observe stochastic demands for their own product only. Collusion is hard to sustain in such an environment as no coordination device exists for punishing "secret price-cutting." To see the problem, consider a grim-trigger strategy which reverts to the one-shot Nash equilibrium after a low demand signal. If noisy (but informative) public signals are available, such a strategy can maintain collusion with sufficiently low discounting. Namely, a bad public signal makes simultaneous reversion to punishment possible whether it was caused by secret price-cutting or otherwise. Reversion to punishment is a best response if every other firm is doing the same. Such coordination is clearly impossible based only on noisy private signals. The conclusions of Sekiguchi [23], Bhasker and Van Damme [5], and Mailath and Morris [18] on this class of games all appear to indicate that (approximate) coordination is possible only if those private signals are accurate indicators of other players' action choice or private signals conditional on every action profile.

When players publicly communicate their signals during the course of play, on the other hand, their announcements serve as public signals on which actions can be coordinated. As demonstrated by Kandori and Matsushima [14] and Compte [6, 8], this alternative formulation leads to a much more permissive conclusion. For effective communication, of course, each player must be given a proper incentive to report their signals truthfully. This paper extends the analysis of communication in repeated games with private monitoring when the firms' private signals are correlated conditional on each price profile. Correlation of private signals is a distinguishing feature of this paper and gives rise to a very simple grim-trigger collusion scheme based on a logic entirely different from that used in the above papers.

The collusion scheme considered in this paper is described as follows: Each firm i chooses the collusive price p_i^* during the collusion phase and reports a summary of their private signal at the end of every period. In particular, firm i reports either "0" or "1" depending on whether his signal in that period was below or above a certain threshold m_i^*. Play stays in the collusion phase if and only if the reported signals of all the firms are unanimous (i.e., all "0" or all "1"). Reversion to the punishment phase (with the one-shot

Nash equilibrium) takes place otherwise. If the correlation of private signals is sufficiently high conditional on the price profile $p^* = (p_1^*, \ldots, p_n^*)$, then reversion to punishment is a rare event on the path so that the equilibrium will be almost efficient.

Now consider the firms' incentives. Facing the test described above, a patient firm would try to maximize the probability of unanimous report profiles in the reporting stage. With correlated signals, it is shown that each firm i which has chosen price p_i finds it optimal to report “1” if and only if its signal is higher than some threshold $m_i(p_i)$. In particular, when i has chosen the collusive price p_i^*, the use of the threshold $m_i(p_i) = m_i^*$ as specified by the scheme is optimal. Suppose then that firm i engages in secret price-cutting $p_i < p_i^*$. Since its price cut most likely reduces the demand level of every other firm, firm i may want to report “0” even if its signal is above m_i^* (i.e., set the threshold $m_i(p_i)$ higher than m_i^*) in order to maintain unanimity. It will be shown that no such deviation is profitable if it discontinuously lowers the level of correlation of private signals.

One natural interpretation of the above conditions can be obtained in a model where the demand signal has multiple random components which respond differently to price deviations. In this model, it will be shown that the above conditions on the correlation of private signals hold when idiosyncratic noise terms are small.

Repeated games with imperfect private monitoring have received relatively little attention until recently. Although Stigler's [24] work on repeated Bertrand oligopoly motivates Green and Porter [13] to study repeated games with imperfect monitoring, the latter choose to work with public signals for tractability. Many useful theorems have since been obtained for this case.[1] For games with private monitoring, there exist two distinct approaches. The first approach, which assumes no communication among players, is taken by Bhasker and Van Damme [5], Compte [7], Fudenberg and Levine [11], Lehrer [17], Mailath and Morris [18], and Sekiguchi [23].[2] Although it is largely inconclusive whether or not collusion is sustained without communication, a strong indication is that efficiency requires monitoring to be near perfect or near public. The second approach, which assumes communication,

[1] See, for example, Abreu et al. [1] and Fudenberg et al. [12].

[2] See also Amarante [2].

is first suggested by Matsushima [19].[3] His idea is subsequently developed by Kandori and Matsushima [14] and Compte [6, 8], who each identify informational conditions for folk theorems.[4] As mentioned above, the present paper belongs to the second category. Its informational assumptions, however, are different in nature from those in Kandori and Matsushima [14] or Compte [6, 8]. More discussion on these points is given in Section 6.

The paper is organized as follows: The next section presents a formal model. Section 3 proves the optimality of cutoff reporting under correlated signals. The main theorem of the paper is given in Section 4. Sections 5 applies this theorem to a model in which demand signals consist of multiple random components. Some related issues are discussed in Section 6. Although the discussion of this paper is totally embedded in the Bertrand framework, it will be clear that all the conclusions will hold in any other games that have the same information structure.

2. Model

The set I of n (≥ 2) firms produce and sell products over infinitely many periods. In every period t, firm i chooses price p_i^t from the set R_+ of non-negative real numbers, and then privately observes its own demand $d_i^t \in R_+$ whose probability distribution depends on the price profile $p^t = (p_1^t, \ldots, p_n^t)$ of all firms.[5] Denote the demand profile in period t by $d^t = (d_1^t, \ldots, d_n^t)$. We suppose that $d^1, \ldots, d^t, \ldots$ are independent, and have the identical probability distribution $P(\cdot \mid p)$ conditional on the price profile p.

Firm i's payoff in any period is given by $G_i(p_i, d_i)$ when its own price is p_i and demand is d_i. Consequently, its expected stage-payoff under the price profile p equals $g_i(p) = E[G_i(p_i, d_i) \mid p]$. We assume that the function $g_i : R_+^n \to R$ is bounded (i.e., $\sup_{p \in R_+^n} |g_i(p)| < \infty$), and that the stage-game has a (pure) Nash equilibrium price profile $p^e = (p_1^e, \ldots, p_n^e)$, i.e., $g_i(p_i, p_{-i}^e) \leq g_i(p^e)$ for any $i \in I$ and $p_i \in R_+$. For simplicity, the one-shot equilibrium payoff $g_i(p^e)$ is normalized to zero.

---

[3] The idea of introducing communication into dynamic games is first proposed by Forges [10] and Myerson [21].

[4] See also Ben-Porath and Kahneman [4].

[5] None of the conclusions will be affected if prices are instead assumed to be positive integers.

In the collusion scheme considered in this paper, every firm i is required to publicly report either $r_i^t = 0$ or 1 given its demand signal d_i^t at the end of each period t. Firm i's reporting rule is a measurable mapping $b_i : \mathbb{R}_+^2 \to \{0, 1\}$ which chooses report r_i as a function of its price p_i and signal d_i in the same period. Firm i's (pure) action a_i in each period is the pair (p_i, b_i) of its price and reporting rule. Let B be the set of i's reporting rules and $A = \mathbb{R}_+ \times B$ be the set of its actions.

Firm i's private history after period t is the sequence of its own prices and private signals in periods $1, \ldots, t$. On the other hand, the public history after period t is the sequence of reports from all firms in periods $1, \ldots, t$. Firm i's (pure) strategy is a measurable mapping $\sigma_i : \bigcup_{t=0}^{\infty} (\{0,1\}^n \times \mathbb{R}_+^2)^t \to A$ which chooses the action as a function of its private history as well as the public history. Firm i's strategy is public if it depends only on the public history and not on its private history.[6] Let $\delta \in [0, 1)$ be the common discount factor of the firms. Given the strategy profile $\sigma = (\sigma_1, \ldots, \sigma_n)$, firm i's average payoff $V_i(\sigma, \delta)$ in the repeated game is defined in the usual manner. A (Nash) equilibrium of the repeated game is a strategy profile σ such that $V_i(\sigma, \delta) \geq V_i(\sigma_i', \sigma_{-i}, \delta)$ for every σ_i' and $i \in I$. An equilibrium σ is public if each σ_i is public. A public equilibrium is perfect if every continuation strategy profile after any public history is again a (public) equilibrium.[7]

This paper analyzes a simple grim-trigger public equilibrium σ^* with the collusion and punishment phases. It is characterized by three sets of parameters $p^* = (p_1^*, \ldots, p_n^*) \in \mathbb{R}_+$, $T \in \mathbb{N}$, and $m^* = (m_1^*, \ldots, m_n^*) \in \mathbb{R}_+^n$. p^* is the price profile to be sustained in the collusion phase, and satisfies $g_i^* = g_i(p^*) > 0$ for each $i \in I$. For example, it can be the price vector that maximizes the collusive profit $\sum_{i=1}^n g_i(p)$ over $p \in \mathbb{R}_+^n$. T is the cycle of the game as explained below. m_i^* is the threshold that firm i is supposed to use in the reporting of its private signals in the collusion phase: it should report $r_i^t = 1$ if $d_i^t \geq m_i^*$ and $r_i^t = 0$ if $d_i^t < m_i^*$.

We suppose that σ^* is T-segmented in the sense that it divides the repeated game into T separate component games that are independent of each other.[8] Specifically, the

[6] Note that a firm's report can depend on the current private signal even if it uses a public strategy.
[7] See Fudenberg et al. [12].
[8] This construction follows that of Ellison [9]. This technique is also used in Sekiguchi [23].

t^{th} component game ($1 \le t \le T$) consists of periods $t; T + t; 2T + t; \ldots$, and the reported signals in any period only affect the continuation play in the same component game. Within each component game, σ^* is the standard grim-trigger strategy profile: It starts with the collusion phase and stays there if and only if report profile r is unanimous in the sense that $r_1 = \cdots = r_n$. It will revert to the punishment phase otherwise. Firm i's strategy σ^*_i chooses p^*_i and reports signal d_i based on the threshold m^*_i as specified above in the collusion phase. In the punishment phase, it chooses the one-shot Nash equilibrium price p^e_i.

It is clear that the firms face no incentive problem in the punishment phase. The numbers T and m^* will be chosen so that it is optimal for every firm to choose p^*_i and report truthfully in the collusion phase.

3. Correlated Signals and Cutoff Reporting

The threshold m^*_i for reporting by firm i is specified so that if every other firm j uses threshold m^*_j, the probability of unanimous report profiles under the price profile p^* is maximized when firm i also uses the threshold m^*_i. Formally, we assume that there exists $m^* = (m^*_1; \ldots; m^*_n) \in (0; 1)^n$ such that

$$\text{(1a)} \qquad P(\min_{j \in I}(d_j - m^*_j) \ge 0 \mid p^*) > 0; \quad P(\max_{j \in I}(d_j - m^*_j) < 0 \mid p^*) > 0;$$

and for every $i \in I$,

$$\text{(1b)} \qquad m^*_i \in \arg\max_{m_i \in R_+} \Big\{ P(\min_{j \neq i}(d_j - m^*_j) \ge 0; d_i \ge m_i \mid p^*) + P(\max_{j \neq i}(d_j - m^*_j) < 0; d_i < m_i \mid p^*) \Big\}:$$

In general, it is difficult to establish the existence of m^* that satisfies these conditions.[9] If $n = 2$, or if the distribution $P(\cdot \mid p^*)$ is symmetric, however, it can be verified that some mild regularity conditions on P guarantee the existence of such thresholds.[10] Based

[9]Note that (1b) alone would be trivially satisfied if m^*_i is a lower or upper bound of the support of d_i for each $i \in I$. (1a) prevents this type of choice. On the other hand, any interior solution to the first-order conditions can be shown to satisfy (1) under a condition similar to Assumption 1 below. In this case, numerical computation of m^* from the first-order conditions is possible.

[10]See Section 5 for a discussion of the symmetric case.

on these $m_1^*, \ldots, m_n^*$, the demand levels are assumed to be “positively correlated” across firms in the following sense:

Assumption 1: For each $i \in I$ and $p_i \in \mathbb{R}_+$, there exists $m_i(p_i) \in [0, \infty]$ such that

$$P(\min_{j \neq i}(d_j - m_j^*) \geq 0 \mid d_i, p_i, p_{-i}^*) \geq P(\max_{j \neq i}(d_j - m_j^*) < 0 \mid d_i, p_i, p_{-i}^*)$$

for $P(\cdot \mid p_i, p_{-i}^*)$-a.e. $d_i \geq m_i(p_i)$, and

$$P(\min_{j \neq i}(d_j - m_j^*) \geq 0 \mid d_i, p_i, p_{-i}^*) \leq P(\max_{j \neq i}(d_j - m_j^*) < 0 \mid d_i, p_i, p_{-i}^*)$$

for $P(\cdot \mid p_i, p_{-i}^*)$-a.e. $d_i < m_i(p_i)$.

It can be seen that Assumption 1 is a single-crossing property of the conditional probabilities. In particular, Assumption 1 holds if the demand profile $d = (d_1, \ldots, d_n)$ is affiliated given each price profile (p_i, p_{-i}^*) since then $P(\min_{j \neq i}(d_j - m_j^*) \geq 0 \mid d_i, p_i, p_{-i}^*)$ (resp. $P(\max_{j \neq i}(d_j - m_j^*) < 0 \mid d_i, p_i, p_{-i}^*)$) is an increasing (resp. decreasing) function of d_i.[11]

We set $m_i(p_i^*) = m_i^*$ when $p_i = p_i^*$. This is justified as follows: Suppose that $m_i(p_i^*) > m_i^*$. Note that the probability of unanimous report profiles can be written as

$$\begin{aligned} & P(\min_{j \neq i}(d_j - m_j^*) \geq 0, d_i \geq m_i \mid p^*) + P(\max_{j \neq i}(d_j - m_j^*) < 0, d_i < m_i \mid p^*) \\ = & \int_0^{m_i} P(\max_{j \neq i}(d_j - m_j^*) < 0 \mid d_i, p^*)\, dP(d_i \mid p^*) \\ + & \int_{m_i}^{m_i(p_i^*)} P(\min_{j \neq i}(d_j - m_j^*) \geq 0 \mid d_i, p^*)\, dP(d_i \mid p^*) \\ + & \int_{m_i(p_i^*)}^{\infty} P(\min_{j \neq i}(d_j - m_j^*) \geq 0 \mid d_i, p^*)\, dP(d_i \mid p^*). \end{aligned}$$

The fact that this quantity is maximized when $m_i = m_i^*$ by (1), as well as the definition of $m_i(p_i^*)$, implies that

$$P(\min_{j \neq i}(d_j - m_j^*) \geq 0 \mid d_i, p^*) = P(\max_{j \neq i}(d_j - m_j^*) < 0 \mid d_i, p^*) \text{ for a.e. } d_i \in [m_i^*, m_i(p_i^*)).$$

[11] The conditional probabilities for affiliated distributions are interpreted as the regular version derived from the densities. For the discussion of affiliation, see Milgrom and Weber [20], and Karlin and Rinott [16].

Thus any number in the interval $[m_i^*, m_i(p_i^*))$ also qualifies as $m_i(p_i^*)$, and in particular, we can take $m_i(p_i^*) = m_i^*$. The same is true when $m_i(p_i^*) < m_i^*$. Of course, $m_i(p_i^*) = m_i^*$ must hold when $m_i(p_i^*)$ is uniquely determined.

Let $\hat{b}_i \in B$ be the reporting rule defined by

$$\hat{b}_i(p_i, d_i) = \begin{cases} 1 & \text{if } d_i \geq m_i(p_i), \\ 0 & \text{otherwise.} \end{cases}$$

Namely, $\hat{b}_i$ is the cutoff reporting rule with the threshold $m_i(p_i)$. We suppose that firm i's public strategy σ_i^* described in the previous section plays the pair $a_i^* = (p_i^*, \hat{b}_i)$ in the collusion phase, and $a_i^e = (p_i^e, \hat{b}_i)$ in the punishment phase. With slight abuse of notation, we let $P(\cdot \mid a)$ denote the probability distribution of the report profile r under the action profile $a = (a_1, \ldots, a_n)$. Theorem 1 below states that the reporting rule $\hat{b}_i$ maximizes the probability of unanimous profiles (and hence the likelihood of staying in the collusion phase) regardless of firm i's price choice. For each report profile $r = (r_1, \ldots, r_n) \in \{0, 1\}^n$, let $s(r) = 0$ if r is unanimous in the sense that $r_1 = \cdots = r_n$, and $s(r) = 1$ otherwise.

Lemma 1. Suppose that Assumption 1 holds at p^*. Then for any $i \in I$, $p_i \in \mathbb{R}_+$ and $b_i \in B$,

$$P(s(r) = 0 \mid p_i, b_i, a_{-i}^*) \leq P(s(r) = 0 \mid p_i, \hat{b}_i, a_{-i}^*).$$

Proof: See the Appendix.

By Lemma 1, it suffices to check the profitability of (one-step) deviations of the form $(p_i, \hat{b}_i)$.

4. Efficient Collusion

Let

$$\alpha = P(\min_{j \in I}(d_j - m_j^*) < 0 \leq \max_{j \in I}(d_j - m_j^*) \mid p^*)$$

be the probability that the report profile is not unanimous when every firm plays $a_i^* = (p_i^*, \hat{b}_i)$. Similarly, let

$$\beta_i(p_i) = P(\min_{j \neq i}(d_j - m_j^*) < 0, d_i \geq m_i(p_i) \mid p_i, p_{-i}^*)$$
$$+ P(\max_{j \neq i}(d_j - m_j^*) \geq 0, d_i < m_i(p_i) \mid p_i, p_{-i}^*)$$

be the probability of the same event when firm i unilaterally deviates to price p_i and uses the reporting rule $\hat{b}_i$ (i.e., uses threshold $m_i(p_i)$ in reporting d_i). Assumption 2 below asserts that no price deviation, whether profitable in the short-run or not, lowers the probability of non-unanimous report profiles.

Assumption 2: For each $i \in I$, $\alpha \leq \inf_{p_i \in R_+} \beta_i(p_i)$.

Since no deviation $(p_i, \hat{b}_i)$ such that $g_i(p_i, p^*_{-i}) \leq g^*_i$ will increase firm i's overall payoff by Assumption 2, it suffices to consider a deviation $(p_i, \hat{b}_i)$ which strictly increases the stage payoff. Let

$$\beta_i = \inf\{\beta_i(p_i) : g_i(p_i, p^*_{-i}) > g^*_i\}.$$

Let $v_i(\delta) = V_i(\sigma^*, \delta)$ denote firm i's overall average payoff in the repeated game under the T-segmented grim-trigger strategy profile σ^* described earlier. Since $v_i(\delta)$ equals firm i's average payoff in each component game, it satisfies the following recursive equation:

$$v_i(\delta) = (1 - \delta^T) g^*_i + \delta^T P(s(r) = 0 \mid a^*)\, v_i(\delta).$$

Noting $P(s(r) = 0 \mid a^*) = 1 - \alpha$, we can solve the above equation to obtain

$$v_i(\delta) = \frac{(1 - \delta^T)\, g^*_i}{1 - \delta^T (1 - \alpha)}. \tag{2}$$

Consider next a one-step deviation $(p_i, \hat{b}_i)$ such that $g_i(p_i, p^*_{-i}) > g^*_i$ in any period during the collusion phase of any component game. Since firm i's effective discount factor is δ^T, no such deviation is profitable if

$$(1 - \delta^T)\, \bar{g}_i + \delta^T P(s(r) = 0 \mid p_i, \hat{b}_i, a^*_{-i})\, v_i(\delta) \leq (1 - \delta^T)\, g^*_i + \delta^T P(s(r) = 0 \mid a^*)\, v_i(\delta),$$

where $\bar{g}_i = \sup_{p_i \in R_+} g_i(p_i, p^*_{-i})$. Note in particular that the above inequality is independent of whether other component games are in the collusion phase or not. Since $P(s(r) = 0 \mid p_i, \hat{b}_i, a^*_{-i}) \leq 1 - \beta_i$ by definition, the above inequality is implied by

$$\frac{\delta^T}{1 - \delta^T} (\beta_i - \alpha)\, v_i(\delta) \geq \bar{g}_i - g^*_i. \tag{3}$$

Theorem 1 below describes the conditions on α and β_i which ensure the existence of an almost efficient equilibrium for sufficiently patient firms. In particular, the desired conclusion holds if α is sufficiently small in absolute terms as well as when compared to β_i.

Theorem 1. Suppose that Assumptions 1 and 2 hold and let $\varepsilon > 0$ be any small number. If α and β_i satisfy

$$\min_{i \in I} \frac{(\beta_i - \alpha)(g_i^* - \varepsilon)}{\bar{g}_i - g_i^*} > \max_{i \in I} \frac{\alpha(g_i^* - \varepsilon)}{\varepsilon}; \tag{4}$$

then there exists $\underline{\delta} < 1$ such that for any $\delta > \underline{\delta}$, the T-segmented grim-trigger strategy profile σ^* is a perfect public equilibrium for some T and yields payoff $v_i(\delta) = V_i(\sigma^*; \delta) > g_i^* - \varepsilon$ for every $i \in I$.

Proof: Note that (4) implies

$$\max_{i \in I} \frac{\bar{g}_i - g_i^*}{(\beta_i - \alpha)(g_i^* - \varepsilon) + \bar{g}_i - g_i^*} < \min_{i \in I} \frac{\varepsilon}{\alpha(g_i^* - \varepsilon) + \varepsilon}.$$

Take $\underline{\delta} < 1$ large enough so that for any $\delta > \underline{\delta}$, there exists $T \in \mathbb{N}$ such that

$$\max_{i \in I} \frac{\bar{g}_i - g_i^*}{(\beta_i - \alpha)(g_i^* - \varepsilon) + \bar{g}_i - g_i^*} \leq \delta^T < \min_{i \in I} \frac{\varepsilon}{\alpha(g_i^* - \varepsilon) + \varepsilon}.$$

The right inequality and (2) imply $v_i(\delta) > g_i^* - \varepsilon$, while the left inequality implies (3). //

5. Demand Functions with Multiple Random Components

In this section, the above theorem is applied to a symmetric model in which the demand signal d_i is expressed as the sum of three non-negative random variables u, w and z_i as follows:

$$d_i = q_i(p)\,u + w + \rho z_i; \tag{5}$$

where $q_i : \mathbb{R}^n_+ \to \mathbb{R}_+$ is a (deterministic) function of the price profile p, and $\rho > 0$ is a constant. Note that u and w are common for every i, while z_i is idiosyncratic to each firm. One interpretation is that u (resp. w) describes the behavior of "global" consumers who regard the products of the n firms as substitutes (resp. complements) because of their technology or taste, while z_i captures "local" consumers for whom the n products are highly differentiated. For concreteness, suppose that the function q_i is strictly decreasing in p_i and strictly increasing in p_j $(j \neq i)$. For symmetry, we require that $q_i(p) = q_j(p')$ for any $i, j \in I$ and p, p' such that $p_i = p'_j$, $p_j = p'_i$, and $p_{-i-j} = p'_{-i-j}$. The assumption that w or z_i does not depend on price is purely for simplicity.[12] We assume that u and w are independent and have the density functions f_u and f_w, respectively. The idiosyncratic terms $z_1, \ldots, z_n$ are independent of one another and of u and w, and have the identical density function f_z. In accordance with the assumption that u, w and z_i are all non-negative, f_u, f_w and f_z all equal zero on $(-\infty, 0)$. It is also assumed that each one of them is strictly positive and continuous on $\mathbb{R}_+$.

The following property of a density function implies the positive correlation of demand signals in the sense of Assumption 1: A density function f on $\mathbb{R}$ is a Polya function of degree 2 (PF_2, Karlin [15]) if

$$\text{(6)} \qquad f(x_2 - y_2)\, f(x_1 - y_1) \geq f(x_2 - y_1)\, f(x_1 - y_2) \quad \text{for any } x_1 < x_2 \text{ and } y_1 < y_2.$$

For example, any Gamma distribution, including the exponential distributions, is PF_2. More generally, it can be verified from (6) that the density function f of a non-negative random variable is PF_2 if for any $y > 0$,

$$\frac{f(x+y)}{f(x)} \text{ is (weakly) decreasing in } x \in \mathbb{R}_+.$$

In what follows, it is assumed that f_u, f_w and f_z are all PF_2.

Lemma 2. *Suppose that the demand signals $(d_1, \ldots, d_n)$ are generated according to (5). If f_u, f_w and f_z are all PF_2, then Assumption 1 holds for any m^* and p^*.*

Proof: See the Appendix.

Suppose that the firms attempt to sustain a symmetric price profile $p^* = (p_1^*, \ldots, p_n^*)$, $p_1^* = \cdots = p_n^*$, such that $q^* = q_i(p^*) > 0$. Let $m_1^* = \cdots = m_n^*$ be the common threshold for reporting as defined by (1). Note that such a threshold indeed exists: Under the above assumption on the distribution, it can be checked that m^* satisfying (1) can be obtained as a solution to the following equation of m_i:

$$P(\min_{j \neq i} d_j \geq m_i \mid d_i = m_i; p^*) - P(\max_{j \neq i} d_j \geq m_i \mid d_i = m_i; p^*) = 0;$$

[12] See the discussion after Theorem 2.

where the conditional probabilities are understood to be the regular version derived from the densities. The above equation can be rewritten as:

$$\int_{[m_i,\infty)^{n-1}} dP(d_{-i} \mid d_i = m_i, p^*) - \int_{[0,m_i)^{n-1}} dP(d_{-i} \mid d_i = m_i, p^*) = 0.$$

The left-hand side is continuous in m_i by the above assumptions on the distribution, and equals 1 when $m_i = 0$ and -1 when $m_i = \infty$. Therefore, the intermediate value theorem implies that there exists $m_i = m_i^*$ which solves this equation.

The last assumption concerns the functional form of q_i. As is usually the case in Bertrand models, a small price deviation by any firm i from the symmetric price profile p^* is assumed to have a "discontinuous" effect on the behavior of $q_i(p_i, p^*_{-i})$ and $q_j(p_i, p^*_{-i})$. Specifically, assume that there exists $\kappa > 1$ such that for any $i \neq j$,

$$\text{(7)} \qquad \frac{q_i(p_i, p^*_{-i})}{q_j(p_i, p^*_{-i})} > \kappa \text{ for any } p_i < p_i^*, \text{ and } \frac{q_i(p_i, p^*_{-i})}{q_j(p_i, p^*_{-i})} < \frac{1}{\kappa} \text{ for any } p_i > p_i^*,$$

where the fraction is interpreted as ∞ if the denominator is zero. Namely, if firm i lowers its price slightly from p^*, then the price-dependent component of its own demand jumps up compared to that of every other firm. If firm i raises its price slightly, on the other hand, the price-dependent component of its own demand jumps down compared to that of every other firm. Note that this is a local condition around p^*, and satisfied in the standard Bertrand model in which $q_i(p)$ itself is the demand function. It is also naturally satisfied if prices are restricted to positive integers.

Theorem 2. Suppose that the demand signals $(d_1, \dots, d_n)$ are generated according to (5), that u, w and z have PF_2 densities, and that $(q_1, \dots, q_n)$ satisfies (7). Then for any $\varepsilon > 0$, there exists $\bar{\rho} > 0$ such that the following is true if $\rho < \bar{\rho}$: There exists $\underline{\delta} < 1$ such that if $\delta > \underline{\delta}$, then the T-segmented grim-trigger strategy profile σ^* is a perfect public equilibrium for some T and yields payoff $v_i(\delta) > g_i^* - \varepsilon$ for every $i \in I$.

Proof: See the Appendix.

Theorem 2 can be illustrated as follows: Suppose for simplicity that $\rho = 0$. In this case, if firm i chooses p_i^* and reports truthfully, then every firm receives exactly the same signal so that the probability α of non-unanimous profiles on the path is zero. On the other hand, suppose that firm i secretly cuts its price to p_i. If a unanimous report is to be achieved, firm i needs to estimate the value of $q_j u + w$ conditional on the observation of $q_i u + w$. Since neither u nor w is directly observed, a range of possibilities must be allowed as to the values of these random variables. This, however, results in the loss of accuracy. To see this, suppose for simplicity that there are only two firms i and j. The probability of non-unanimous profiles is then given by

$$\beta_i = P(q_i u + w \geq m_i, q_j u + w < m_j^*) + P(q_i u + w < m_i, q_j u + w \geq m_j^*).$$

Figure 1 depicts the corresponding areas when $m_i/m_j^* \in (1, q_i/q_j)$. Note by (7) that the ratio of coefficients of u in d_i and d_j changes discontinuously in response to any price deviation by i so that the two lines in the figure have significantly different slopes for any $p_i \neq p_i^*$. Consequently, firm i cannot make the probability β_i arbitrarily small by simply adjusting the threshold m_i.

It should be noted that the above discussion also suggests that the same conclusion holds when the demand function is instead given by $d_i = s_i(p)u + t_i(p)w + \rho z_i$ for some function $t_i : R_+^n \to R_+$, as long as s_i and t_i behave sufficiently differently from each other (around p^*) for any price deviation by i.[13]

6. Discussions

Theorem 1 requires the correlation of private signals to be high on the equilibrium path, and strictly lower off the path. This condition should be contrasted with the informational assumptions used in the models of private monitoring without communication (Sekiguchi [23], Bhasker and Van Damme [5], and Mailath and Morris [18]). In these models, monitoring is either nearly perfect in that a player's private signal accurately reflects other players' action choice, or nearly public in that it accurately reflects their private signals, both conditional on every action profile. High correlation of private signals on the path in the current model is close to the assumption of near public monitoring conditional on the particular profile p^*.[14] Lower correlation off the path, on the other hand, implies

[13] Specifically, we need to have $\frac{s_i(p_i, p^*_{-i})}{s_j(p_i, p^*_{-i})} \frac{t_j(p_i, p^*_{-i})}{t_i(p_i, p^*_{-i})}$ bounded away from one for any $p_i \neq p_i^*$.

[14] Note, however, that high correlation relative to thresholds $m_1^*, \ldots, m_n^*$ is much weaker than near public monitoring.

that a player's private signal is less accurate as an indicator of others' private signals conditional on any profile $(p_i; p^*_{-i})$ $(p_i \neq p^*_i)$. It is this gap in the levels of accuracy that this collusion scheme exploits.

The informational assumptions of this paper are also different in nature from the "distinguishability conditions" used by Kandori and Matsushima [14] and Compte [6]. The distinguishability conditions require that given any action profile and any pair of players i and j, i's deviation be statistically distinguishable from j's deviation when private signals of players other than i and j are jointly evaluated.[15] Clearly, these conditions require the existence of three or more players. They also require the finiteness of the action set. Their generalization to games with infinite actions, such as the standard Bertrand game, is not straightforward.

In some cases, it is not inconceivable that firms spy on their competitors' prices in an effort to facilitate collusion. The paper's conclusion, however, suggests that they may instead agree to use a much simpler scheme which makes such monitoring activities redundant. It remains open whether communication during the course of play is really necessary to sustain collusion. It should be noted, however, that even in models without explicit communication, some form of communication before the game is implicitly assumed. Otherwise, the players have difficulty agreeing on a particular equilibrium. Although attempts by competing firms to exchange information through meetings such as trade associations have been often found illegal under Sherman Act (Scherer [22, Section 19]), there appear to be many other less conspicuous ways of communicating one's private signals to its competitors. Bidders' using the last few digits of bids as a communication device in the Federal spectrum auctions is a well-known example. As another example, we can ask the motive behind airline companies' large newspaper advertisements with detailed fares which are not necessarily honored.[16]

With the correlation of private signals, it is possible to consider an alternative scheme which requires each firm i to report its raw demand signal d_i and suggests reversion to

[15] To be more precise, this must be true at every action profile which corresponds to an extreme point of the feasible payoff set. This description of the conditions is based on Kandori and Matsushima [14]. Compte's [6] conditions are similar but slightly different.

[16] This observation was made and suggested by Jack Ochs.

punishment if and only if the discrepancy between the maximum and minimum of those reported numbers exceeds some threshold. This approach, however, runs into some difficulty. In our formulation, the number of viable deviations can be effectively limited by the consideration that for any deviation in reporting, there exists an alternative cutoff reporting rule that is at least as good (Lemma 1). In this alternative scheme, on the other hand, it is difficult to identify such a natural class of deviations. This makes the verification of the incentive conditions much more difficult.

Appendix

Proof of Lemma 1: Note that

$$
\begin{aligned}
& P(s(r)=0 \mid d_i; p_i; b_i; a^*_{-i}) \\
&= P(\min_{j\neq i}(d_j - m_j^*) \geq 0 \mid p_i; d_i; p^*_{-i})\, b_i(d_i) \\
&\quad + P(\max_{j\neq i}(d_j - m_j^*) < 0 \mid p_i; d_i; p^*_{-i})\, \{1 - b_i(d_i)\} \\
&\leq P(\min_{j\neq i}(d_j - m_j^*) \geq 0 \mid p_i; d_i; p^*_{-i})\, \hat{b}_i(d_i) \\
&\quad + P(\max_{j\neq i}(d_j - m_j^*) < 0 \mid p_i; d_i; p^*_{-i})\, \{1 - \hat{b}_i(d_i)\} \\
&= P(s(r)=0 \mid d_i; p_i; \hat{b}_i; a^*_{-i}) \quad \text{a.s.},
\end{aligned}
$$

where the inequality follows from the facts that $\hat{b}_i(d_i) = 1$ if and only if $d_i \geq m_i(p_i)$, and that $d_i \geq m_i(p_i)$ implies

$$
P(\min_{j\neq i}(d_j - m_j^*) \geq 0 \mid p_i; d_i; p^*_{-i}) \geq P(\max_{j\neq i}(d_j - m_j^*) < 0 \mid p_i; d_i; p^*_{-i}) \quad \text{a.s.},
$$

while $d_i < m_i(p_i)$ implies

$$
P(\min_{j\neq i}(d_j - m_j^*) \geq 0 \mid p_i; d_i; p^*_{-i}) \leq P(\max_{j\neq i}(d_j - m_j^*) < 0 \mid p_i; d_i; p^*_{-i}) \quad \text{a.s.}
$$

by Assumption 1. It follows that

$$
\begin{aligned}
P(s(r)=0 \mid p_i; b_i; a^*_{-i}) &= \int_{R^2_+} P(s(r)=0 \mid d_i; p_i; b_i; a^*_{-i})\, dP(d_i \mid p_i; p^*_{-i}); \\
&\leq \int_{R^2_+} P(s(r)=0 \mid d_i; p_i; \hat{b}_i; a^*_{-i})\, dP(d_i \mid p_i; p^*_{-i}); \\
&= P(s(r)=0 \mid p_i; \hat{b}_i; a^*_{-i}):
\end{aligned}
$$

Therefore, replacing b_i by $\hat{b}_i$ increases the probability of a unanimous report profile. //

Proof of Lemma 2: For any integer $l \geq 1$, a non-negative function $K : \mathbb{R}^l \to \mathbb{R}_+$ is said to be multivariate totally positive of order 2 (MTP$_2$, Karlin and Rinott [16]) if

$$K(x \wedge y)\, K(x \vee y) \geq K(x)\, K(y) \quad \text{for every } x, y \in \mathbb{R}^l.$$

Given any price p_i, write $q_j = q_j(p_i, p^*_{-i})$ for $j \in I$ and $y_i = q_i u + w$ and $y_j = q_j u + w$ ($j \neq i$). Suppose first that $p_i \neq p^*_i$ so that $q_i \neq q_j$ if $j \neq i$. By the standard argument, the joint density of y_i and y_j can be calculated as

$$\varphi(y_i, y_j) = f_u\left(\frac{y_i - y_j}{q_i - q_j}\right) f_w\left(\frac{q_i y_j - q_j y_i}{q_i - q_j}\right) \frac{1}{|q_i - q_j|}.$$

If we define $K_u(y_i, y_j) = f_u\left(\frac{y_i - y_j}{q_i - q_j}\right)$ and $K_w(y_i, y_j) = f_w\left(\frac{q_i y_j - q_j y_i}{q_i - q_j}\right)$, then it can be verified that both K_u and K_w are MTP$_2$ since f_u and f_w are PF$_2$. Since the product of MTP$_2$ functions is again MTP$_2$ (Karlin and Rinott [16, Proposition 3.2]), it follows that φ itself is MTP$_2$. Note now that the joint density of $d_i = y_i + \rho z_i$ and y_j is given by

$$\psi(d_i, y_j) = \int_0^\infty \varphi(s, y_j)\, f_z\left(\frac{d_i - s}{\rho}\right) ds.$$

Note that $K_{y_i y_j}(d_i, y_i, y_j) = \varphi(y_i, y_j)$ is MTP$_2$ by the above discussion. Since f_z is PF$_2$, $K_z(d_i, y_i, y_j) = f_z\left(\frac{d_i - y_i}{\rho}\right)$ is MTP$_2$ as well. Therefore, the integrand is also MTP$_2$, and so is ψ (Karlin and Rinott [16, Propositions 3.3]).

Suppose next that $p_i = p^*_i$ so that $q_i = q_j$. The joint density of (d_i, y_j) is given by

$$\psi(d_i, y_j) = \int_0^\infty f_u(s)\, f_w(y_j - q_i s)\, f_z\left(\frac{d_i - s}{\rho}\right) \frac{1}{\rho}\, ds.$$

Since $L_u(d_i, y_j, u) = f_u(u)$, $L_w(d_i, y_j, u) = f_w(y_j - q_i u)$ and $L_z(d_i, y_j, u) = f_z\left(\frac{d_i - u}{\rho}\right)$ are all MTP$_2$, it follows that ψ is MTP$_2$ by the same logic as above.

We now show that under any price profile (p_i, p^*_{-i}), $P(\min_{k \neq i} d_k \geq m^*_i \mid d_i; p_i, p^*_{-i})$ is an increasing function of d_i. Note that

$$\begin{aligned} P(\min_{k \neq i} d_k \geq m^*_i \mid d_i; p_i, p^*_{-i}) &= P(y_j + \min_{k \neq i} z_k \geq m^*_i \mid d_i; p_i, p^*_{-i}) \\ &= \int_{\mathbb{R}_+^{n-1}} P(y_j \geq m^*_i - \min_{k \neq i} z_k \mid d_i; p_i, p^*_{-i})\, f_{z_{-i}}(z_{-i})\, dz_{-i}. \end{aligned}$$

Since $\psi(d_i; y_j)$ is MTP_2, the integrand is an increasing function of d_i (Karlin and Rinott [16, Theorem 4.1]). Hence, the desired conclusion follows. A similar argument shows that $P(\max_{j \neq i} d_j < m_i^* \mid d_i; p_i; p_{-i}^*)$ is a decreasing function of d_i. These imply Assumption 1. //

Proof of Theorem 2: Let α and β ($\equiv \beta_1 = \cdots = \beta_n$) be indexed by $\rho > 0$. It is shown below that (A) $\lim_{\rho \to 0} \alpha(\rho) = 0$, and that (B) there exist $\gamma > 0$ and $\bar{\rho} > 0$ such that $\beta(\rho) > \gamma$ if $\rho < \bar{\rho}$. These imply that (4) holds for a sufficiently small $\rho > 0$, and hence the desired conclusion follows. The analysis below concentrates on the case where $p_i < p_i^*$. The other case $p_i > p_i^*$ can be treated in the symmetric manner.

(A) $\lim_{\rho \to 0} \alpha(\rho) = 0$.

Take any $\varepsilon > 0$. For any fixed u' and $z_1'; \ldots; z_n'$, the Lebesgue measure of the interval

$$\left[m_i^* - q^* u' - \rho \max_{j \in I} z_j';\; m_i^* - q^* u' - \rho \min_{j \in I} z_j' \right)$$

is equal to $\rho(\max_{j \in I} z_j' - \min_{j \in I} z_j')$. Since w is absolutely continuous, there exists $\delta > 0$ such that for any u', $z_1' \ldots; z_n'$, if $\rho(\max_{j \in I} z_j' - \min_{j \in I} z_j') < \delta$, then

$$P(m_i^* - q^* u' - \rho \max_{j \in I} z_j' \leq w < m_i^* - q^* u' - \rho \min_{j \in I} z_j') < \varepsilon.$$ [17]

Therefore, in view of independence, we have

$$\begin{aligned}
\alpha(\rho) = & \int_{R_+ \times [0; \frac{\delta}{\rho})^n} P(m_i^* - q^* u - \rho \max_{j \in I} z_j \leq w < m_i^* - q^* u - \rho \min_{j \in I} z_j)\, dP(u; z \mid p^*) \\
& + \int_{R_+ \times (R_+^n \setminus [0; \frac{\delta}{\rho})^n)} P(m_i^* - q^* u - \rho \max_{j \in I} z_j \leq w < m_i^* - q^* u - \rho \min_{j \in I} z_j)\, dP(u; z \mid p^*) \\
& \leq \varepsilon \int_{[0; \frac{\delta}{\rho})^n} dP(u; z \mid p^*) + \int_{R_+^n \setminus [0; \frac{\delta}{\rho})^n} dP(u; z \mid p^*):
\end{aligned}$$

Since the last quantity can be made smaller than 2ε when ρ is small enough, and since ε is arbitrary, we conclude that $\alpha(\rho) \to 0$ as $\rho \to 0$.

(B) There exist $\gamma > 0$ and $\bar{\rho} > 0$ such that $\beta(\rho) > \gamma$ if $\rho < \bar{\rho}$.

---

[17] See Wheeden and Zygmund [25, Theorem 10.34].

For each $\rho > 0$, define $\beta_\rho : \mathbb{R}^2_+ \to \mathbb{R}_+$ by

$$\beta_\rho(p_i, m_i) = P\left[q_i u + w + \rho z_i \ge m_i, \min_{j \neq i}(q_j u + w + \rho z_j) < m_j^*\right] + P\left[q_i u + w + \rho z_i < m_i, \max_{j \neq i}(q_j u + w + \rho z_j) \ge m_j^*\right],$$

where $q_j = q_j(p_i, p^*_{-i})$ for $j \in I$. Likewise, define $\beta_0 : \mathbb{R}^2_+ \to \mathbb{R}_+$ by

$$\beta_0(p_i, m_i) = P\left[q_i u + w \ge m_i, q_j u + w < m_j^*\right] + P\left[q_i u + w < m_i, q_j u + w \ge m_j^*\right].$$

By symmetry, $q_j = q_l$ if $j, l \neq i$ and $m_1^* = \cdots = m_n^*$ so that j in the definition of β_0 can be replaced by any $l \neq i$ without changing its value. It is clear from the definition that $\beta(\rho) = \inf\{\beta_\rho(p_i, m_i) : p_i < p_i^*, m_i \in \mathbb{R}_+\}$. In what follows, we show that this infimum is strictly positive for ρ small enough in three steps.

i) $\inf\{\beta_0(p_i, m_i) : p_i < p_i^*, m_i \in \mathbb{R}_+\} > 0$.

Writing $\underline{q}_j = \inf\{q_j(p_i, p^*_{-i}) : p_i < p_i^*\}$ and $\bar{q}_j = \sup\{q_j(p_i, p^*_{-i}) : p_i < p_i^*\}$ for $j \in I$, we have

$$\beta_0(p_i, m_i) \ge \varphi(m_i),$$

where $\varphi = \varphi_1 + \varphi_2$, and

$$\varphi_1(m_i) = P\left[\underline{q}_i u + w \ge m_i, \bar{q}_j u + w < m_j^*\right]; \quad \text{and}$$
$$\varphi_2(m_i) = P\left[\bar{q}_i u + w < m_i, \underline{q}_j u + w \ge m_j^*\right].$$

Let κ be as defined in (7). If $m_i \le \kappa m_i^*$, then $\frac{m_i - m_j^*}{\underline{q}_i - \bar{q}_j} < \frac{m_j^*}{\bar{q}_j}$ (> 0), and for any u between these two values, we have $m_i - \underline{q}_i u < m_j^* - \bar{q}_j u$ (> 0). Since u and w both have full support, it then follows that

$$\varphi_1(m_i) \ge P\left(\frac{m_i - m_j^*}{\underline{q}_i - \bar{q}_j} < u < \frac{m_j^*}{\bar{q}_j}, m_i - \underline{q}_i u \le w < m_j^* - \bar{q}_j u\right) > 0 \quad \text{for } m_i \le \kappa m_i^*.$$

On the other hand, if $m_i \ge \kappa m^*$, then for any $u < \frac{m_i - m_j^*}{\bar{q}_i - \underline{q}_j}$ (> 0), we have $m_j^* - \underline{q}_j u < m_i - \bar{q}_i u$ (> 0) so that

$$\varphi_2(m_i) \ge P\left(u < \frac{m_i - m_j^*}{\bar{q}_i - \underline{q}_j}, m_j^* - \underline{q}_j u \le w < m_i - \bar{q}_i u\right) > 0 \quad \text{for } m_i \ge \kappa m_i^*.$$

Note now that ϕ_1 is a decreasing function of m_i while ϕ_2 is an increasing function of m_i. This, together with the above observation, implies that for $m_i \le \kappa m_i^*$,

$$\phi_1(m_i) + \phi_2(m_i) \ge \phi_1(\kappa m_i^*) + \phi_2(m_i) \ge \phi_1(\kappa m_i^*);$$

and that for $m_i \ge \kappa m_i^*$,

$$\phi_1(m_i) + \phi_2(m_i) \ge \phi_1(m_i) + \phi_2(\kappa m_i^*) \ge \phi_2(\kappa m_i^*).$$

The conclusion now follows immediately since

$$\inf_{m_i \in R_+} \phi(m_i) \ge \min\{\phi_1(\kappa m_i^*);\ \phi_2(\kappa m_i^*)\} > 0.$$

ii) $\beta^\rho \to \beta^0$ uniformly over $(p_i; m_i) \in R_+^2$ as $\rho \to 0$.

It can be verified that

$$
\begin{aligned}
&|\beta^\rho(p_i; m_i) - \beta^0(p_i; m_i)| \le 2P(m_i - \rho z_i \le q_i u + w < m_i) \\
\text{(a1)} \qquad &\qquad + P\left(\min_{j \ne i} (q_j u + w + \rho z_j) \ge m_j^*;\ q_j u + w < m_j^*\right) \\
&\qquad + P\left(\max_{j \ne i} (q_j u + w + \rho z_j) \ge m_j^*;\ q_j u + w < m_j^*\right).
\end{aligned}
$$

Take any $\epsilon > 0$. For any u' and z'_i, the Lebesgue measure of the interval $[m_i - q_i u' - \rho z'_i;\ m_i - q_i u')$ is $\rho z'_i$. Since the distribution of w is absolutely continuous, there exists $\eta > 0$ such that for any u' and z'_i, if $\rho z'_i < \zeta$, then

$$P(m_i - q_i u' - \rho z'_i \le w < m_i - q_i u') < \epsilon.$$

Therefore, as in (A) above, we have

$$
\begin{aligned}
P(m_i - \rho z_i \le q_i u + w < m_i) &= \int_{R_+^2} P(m_i - q_i u - \rho z_i \le w < m_i - q_i u)\, dP(u; z_i) \\
&\le \epsilon \int_0^{\eta/\rho} dP(z_i) + \int_{\eta/\rho}^{1} dP(z_i).
\end{aligned}
$$

Note that the last quantity is independent of $(p_i; m_i)$, and can be made smaller than 2ϵ if ρ is sufficiently small. It can be shown through an argument similar to that in (A) that the remaining two terms on the right-hand side of (a1) are bounded above by 2ε each for ρ small enough. Since $\varepsilon > 0$ is arbitrary, we have the desired conclusion.

iii) There exist $\bar{\rho} > 0$ and $\gamma > 0$ such that $\inf\{\beta^{\rho}(p_i; m_i) : p_i < p_i^*; m_i \in R_+\} > \gamma$ for $\rho < \bar{\rho}$.

By (i), there exists $\gamma > 0$ such that $\beta^0(p_i; m_i) > 2\gamma$ for any $(p_i; m_i)$ with $p_i < p_i^*$. By (ii), there exists $\bar{\rho} > 0$ such that $\rho < \bar{\rho}$ implies $|\beta^{\rho}(p_i; m_i) - \beta^0(p_i; m_i)| < \gamma$ for any $(p_i; m_i)$ with $p_i < p_i^*$. Therefore, for any $\rho < \bar{\rho}$, $\beta^{\rho}(p_i; m_i) = \beta^0(p_i; m_i) + \beta^{\rho}(p_i; m_i) - \beta^0(p_i; m_i) > \beta^0(p_i; m_i) - \gamma > \gamma$ for any $(p_i; m_i)$ such that $p_i < p_i^*$. This completes the proof of (B). //

References

1. D. Abreu, D. Pearce, and E. Stacchetti, Toward a theory of discounted repeated games with imperfect monitoring, Econometrica, 58 (1990), 1041-63.
2. M. Amarante, “Discounted repeated games with imperfect private monitoring," mimeo., 1998.
3. M. Aoyagi, “A collusion mechanism in dynamic Bertrand oligopoly under private observation of correlated demand shocks," mimeo., 1998.
4. E. Ben-Porath and M. Kahneman, Communication in repeated games with private monitoring, J. Econ. Theory, 70 (1996), 281-297, doi:10.1006/jeth.1996.0090.
5. V. Bhasker and E. Van Damme, “Moral hazard and private monitoring," mimeo., 1997.
6. O. Compte, Private observations, communication and coordination in repeated games, Ph.D. thesis, Stanford University, Stanford, 1994.
7. O. Compte, “On failing to cooperate when monitoring is private," mimeo., 1996.
8. O. Compte, Communication in repeated games with imperfect private monitoring, Econometrica, 66 (1998), 597-626.
9. G. Ellison, Cooperation in the prisoners' dilemma with anonymous random matching, Rev. Econ. Stud., 61 (1994), 567-588.
10. F. Forges, An approach to communication equilibria, Econometrica, 54 (1986), 1375-1385.

11. D. Fudenberg and D. Levine, An approximate folk theorem with imperfect private information, J. Econ. Theory, 54 (1991), 23-47.

12. D. Fudenberg, D. Levine, and E. Maskin, The folk theorem with imperfect public information, Econometrica, 62 (1994), 997-1039.

13. E. Green and R. Porter, Noncooperative collusion under imperfect public information, Econometrica, 52 (1984), 87-100.

14. M. Kandori and H. Matsushima, Private observation, communication and collusion, Econometrica, 66 (1998), 627-652.

15. S. Karlin, \Total Positivity," Stanford University Press, Stanford, 1968.

16. S. Karlin and Y. Rinott, Classes of orderings of measures and related correlation inequalities. I. Multivariate totally positive distributions, J. Multivariate Analysis, 10 (1980), 467-498.

17. E. Lehrer, Lower equilibrium payo®s in two-player repeated games with nonobservable actions, Int. J. Game Theory, 18 (1989), 57-90.

18. G. Mailath and S. Morris, \Repeated games with almost-public monitoring," mimeo., 1999.

19. H. Matsushima, On the theory of repeated games with private information. Part II: Revelation through communication, Econ. Letters, 35 (1990), 257-261.

20. P. Milgrom and R. Weber, A theory of auctions and competitive bidding, Econometrica, 50 (1982), 1089-1122.

21. R. Myerson, Multistage games with communication, Econometrica, 54 (1986), 323-358.

22. F. M. Scherer, \Industrial Market Structure and Economic Performance," 2nd Ed., Houghton Mi²in, Boston, 1980.

23. T. Sekiguchi, E±ciency in repeated prisoners' dilemma with private monitoring, J. Econ. Theory, 76 (1997), 345-361, doi:10.1006/jeth.1997.2313.

24. G. Stigler, A theory of oligopoly, J. Polit. Econ., 72 (1964), 44-61.

25. R. L. Wheeden and A. Zygmund, \Measure and Integral," Marcel Dekker, New York, 1977.

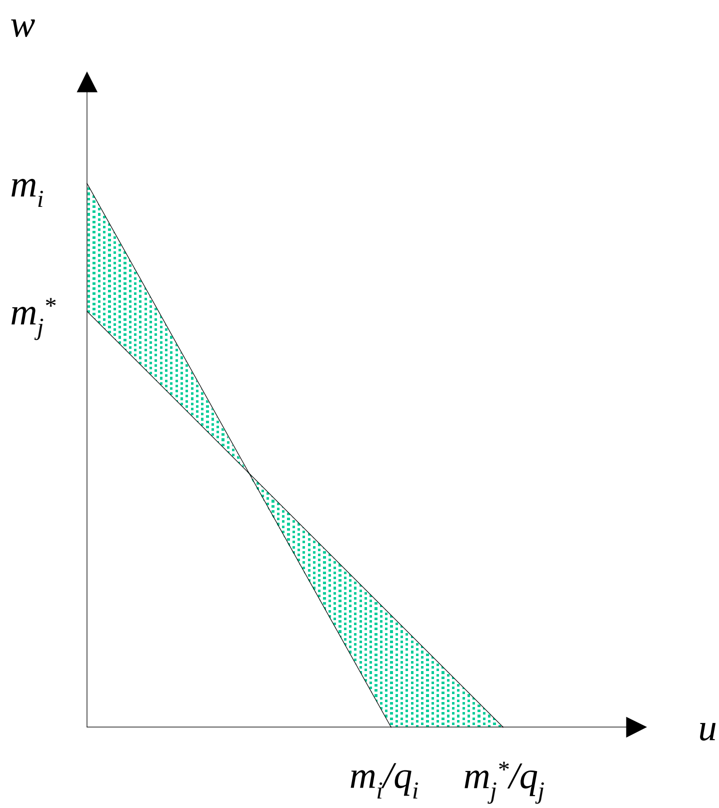

Figure 1