Journal of Economic Theory, 165. 565-600, 2016.

Social Learning and Delay in a Dynamic Model of Price Competition*

Masaki Aoyagi† Manaswini Bhalla‡ Hikmet Gunay§

May 13, 2016

Abstract

This paper studies dynamic price competition between two firms selling differentiated durable goods to two buyers whose valuations of the two goods depend on their own private type as well as that of the other buyer. We derive a key intertemporal property of the equilibrium prices and construct an equilibrium based on this property. We show that social learning reduces the equilibrium prices in the sense that when the buyers are more interdependent and hence have a stronger incentive to wait and see, the firms respond by lowering their period 1 prices. Interestingly, we find that this response by the firms along with the intertemporal property of the equilibrium prices implies that buyers delay their decisions *less* often when they become more interdependent.

Key words: dynamic pricing, delay, social learning, duopoly, product differentiation, durable good, preemption, revenue management, conspicuous consumption.
Journal of Economic Literature Classification Numbers: C72, D82.

1 Introduction

Consumer preferences are inherently interdependent in many durable goods markets. Consider, for example, a potential consumer of a new model of an automobile. Purchase

*We are grateful to two anonymous referees and the editor of the journal whose comments led to a significant improvement of the paper. We are also grateful to Pauli Murto, Juuso Välimäki, and the seminar participants at Singapore Management University for their comments on an earlier version. Aoyagi gratefully acknowledges the financial support from the JSPS (grant numbers: 24653048, 15K13006, 22330061 and 15H03328) and the Joint Usage/Research Center at ISER, Osaka University.

†ISER, Osaka University (aoyagi@iser.osaka-u.ac.jp).
‡Economics and Social Science, Indian Institute of Management, Bangalore (manaswinib@iimb.ernet.in).
§Department of Economics, University of Manitoba (Hikmet.Gunay@umanitoba.ca).

decisions of such a product are accompanied by careful examination of various information collected from a catalog and magazine articles as well as their own experience of products from the same manufacturer. Such information forms the basis of the consumer's *intrinsic valuation* of the product. In many cases, however, consumers don't act on their intrinsic valuations alone and are also concerned about how the product is perceived by other consumers. Behind such a concern may be the presence of conspicuous consumption: Consumers attach higher values to the products that are more highly valued by other consumers.[1] Another reason for the concern may be their awareness that their individual piece of information is imperfect: If they can learn others' information about the product, it will help them form a more accurate estimate of its value to them. In either case, in order to understand consumer behavior in the choice of durable goods, it is important to consider their *extrinsic valuations,* which we define to be the combination of the own intrinsic valuations and the valuations of other consumers.

When direct and truthful communication of private information is not feasible, each consumer has an incentive to wait and see the decisions of other consumers to collect more information. We are interested in the problem of intertemporal price competition between two firms selling differentiated durable goods to such interdependent consumers. In our model, two consumers each have private intrinsic valuations about the the two goods, and buy a single unit of either good in one of the two periods in an irreversible manner. Hence, a consumer in period 1 must decide between buying today from either firm for the quoted price, and waiting until tomorrow. If he waits, he has better information about his valuation, but the price offer by each firm in period 2 is also contingent on the buyers' decision and can be high or low depending on whether its product was chosen in period 1. Each firm, on the other hand, needs to set its price taking into account the consumers' incentives to 'wait and see' as well as the pricing decision of the other firm. For example, by offering a discount in period 1, a firm may preempt the market by capturing one of the consumers and then be able to sell the good to the other consumer at a higher price in period 2. On the other hand, offering a discount may be detrimental to the profits if, for example, it leads to a more intense competition in period 2. Further, each firm needs to take into account the information flow generated by its pricing decision. This simple discussion already suggests the complexity of the strategic interaction between the consumers, between the firms, and between the firms and the consumers.

A more detailed description of our model is as follows: Two firms A and B sell

[1] Conspicuous consumption is an important topic in the marketing literature, and is empirically validated by Wilcox *et al.* (2009), Shukla (2010), and Bian and Forsythe (2012).

differentiated durable goods A and B, respectively, over two periods. There are two consumers $i = 1, 2$ each of whom has an intrinsic valuation for each one of the two goods. We suppose that for each buyer, the relative superiority of good B over good A in terms of his intrinsic valuations is randomly drawn and privately observed, but there is no uncertainty about the average valuations of the two goods. The measure of superiority of B over A is referred to as buyer i's type and denoted s_i. We assume that buyer i's *extrinsic valuation* of each good is the weighted average of his own intrinsic valuation and the other buyer's (intrinsic or extrinsic) valuation of the same good.[2] As a consequence, we can express a buyer's extrinsic valuation as the weighted average of his and the other buyer's types, with the weight placed on the own type larger than that placed on the other buyer's type. Each consumer demands at most one unit of either good and purchases the product at most once in one of the two periods. In period 1, the firms quote prices simultaneously, and the consumers make simultaneous decisions on whether to buy either good or wait until period 2. The public history comprises the prices and consumer decisions in period 1. Given the updated belief about the consumers' types, the firms in period 2 again quote prices simultaneously, and any remaining consumers make purchase decisions simultaneously again based on the updated beliefs about each other's type. The firms and the buyers have a common discount factor, but are assumed to be sufficiently patient for most of the analysis.

Our analysis begins with the observation that in equilibrium, the buyers' period 1 behavior facing any price profile is sorted by their types. Specifically, under any price profile, we show that the type space is divided into three intervals: the buyer types in the lowest interval who have the most favorable intrinsic valuation about good A choose A in period 1, those in the highest interval who have the most favorable intrinsic valuation about good B choose B in period 1, and those in the middle interval who have a moderate intrinsic valuation about both goods defer their decisions until period 2. These intervals are endogenously determined in equilibrium.

The firms' pricing decision in period 2 naturally responds to the buyers' purchasing decisions in period 1. The analysis of an equilibrium requires the identification of the exact relationship between them. Our key result does this by establishing the intertemporal behavior of the critical buyer types in period 1 as described above. Specifically, we prove the *indifference property,* which shows that in equilibrium, the critical buyer type

[2] If the intrinsic valuation reflects noisy information about the value of the product, then the extrinsic valuation would be the weighted average of the intrinsic valuations of the two consumers. In the case of conspicuous consumption, on the other hand, the extrinsic valuation can be the weighted average of the own intrinsic valuation and the extrinsic valuation of the other consumer. Both formulations are consistent with our analysis.

who is indifferent between "buying from firm A (resp. B) in period 1" and "waiting in period 1 and then making a contingent choice in period 2" is also indifferent between "buying from A (resp. B) in period 1" and "waiting in period 1 and then buying from A (resp. B) in period 2 after *any* decision by the other buyer." This property, which holds for any period 1 price pair both on and off the path of play, is extensively used in the derivation and characterization of an equilibrium. We first examine if the equilibrium can be *preemptive* in the sense that any firm that successfully attracts one buyer in period 1 also sells to any remaining buyer in period 2. As will be seen, when the prices are fixed and equal to marginal cost in both periods, buyer behavior is characterized by such preemption. Even when pricing is strategic, such preemption appears plausible: The firm which wins a buyer in period 1 strengthens its position in the period 2 market where the relative value of its own good compared with that of the other good is increased. Despite this intuition, however, we show that no equilibrium entails preemption: Preemption implies fierce competition in the period 2 market over a small interval of active buyer types, and hence low period 2 prices. This in turn implies low period 1 prices through the indifference property, which induce either firm to profitably deviate by increasing its period 1 price.

This finding leads to the consideration of a strategy profile in which the losing firm (if any) in period 1 makes a sale in period 2 with positive probability. The main theorem of the paper uses the indifference property to construct an equilibrium with this property. We observe that the period 1 price in equilibrium entails a discount compared with that in the one-period model to reflect the increased bargaining power of the buyers in the two-period model where they have a delay option. This discount is shown to be increasing in the degree of interdependence of the preferences. We can interpret this as the firms' response to the stronger incentive of the more interdependent consumers to delay their decisions. In other words, social learning imposes a downward pressure on the period 1 prices. Interestingly, however, the lowered period 1 price coupled with the indifference property implies that the probability of delay in equilibrium decreases as the buyers become more interdependent. In contrast, we show that the more interdependent buyers delay more often in the alternative model in which the firms engage in marginal cost pricing in both periods. The buyers' decisions are highly inefficient, and we identify two potential sources of the inefficiency: (i) the distortion in the buyer behavior caused by the change in the period 2 prices in response to their own decisions in period 1, and (ii) the failure on the part of each buyer to internalize the informational externalities that their learning behavior inflicts on the other buyer. Examination of some benchmarks reveals that the first effect is dominant for smaller values of the interdependence parameter, but the second effect becomes more important for larger values.

The paper is organized as follows. After the discussion of the related literature in the next section, we formulate our model in Section 3. Section 4 describes the buyer behavior in period 1, and Section 5 analyzes the period 2 equilibrium. The indifference property is proven in Section 6. We demonstrate the impossibility of the preemptive equilibrium in Section 7, and construct a non-preemptive equilibrium in Section 8. Section 9 analyzes delay in equilibrium and compares its property with that in the model with marginal cost pricing. Comparative statics analysis of efficiency is given in Section 10. We conclude with a discussion in Section 11.[3]

2 Related Literature

Our model extends the standard models of dynamic durable good markets in at least two directions: First, we introduce interdependence in preferences between consumers which we consider essential for many durable goods as discussed above. Second, we introduce competition between the firms as a realistic feature of many durable goods markets.

The assumption on the interdependence of preferences in our model implies the presence of social learning by the consumers. In the social learning literature that begins with Banerjee (1992) and Bikhchandani *et al.* (1992), delay induced by learning is one of the central topics. Among others, Chamley and Gale (1994) and Gul and Lundholm (1995) present a model of strategic delay in the context of dynamic investment decisions.[4] More recently, the literature on social learning looks at the sequential sales of a product of uncertain quality by a monopolist, who optimally controls its price contingent on sales history.[5] The standard assumption there is that each consumer makes a single decision: They either take a price offer, or else exit the market. Our model is new in that it combines the multiple purchase decisions and the strategic pricing of a product. Natural as it may appear, this combination has not been explored before to the best of our knowledge perhaps because of the substantial complications it creates in the technical analysis. In particular, there is fundamental difficulty in checking the firms' deviation incentives in period 1 when those deviations change the buyers' delay incentives and also the outlook of the period 2 market. We show that the problem is solvable with the use of the indifference property mentioned in the Introduction.

The ability of consumers to wait and look for a better opportunity in later periods as

[3]The proofs are either in the Appendix at the end of the paper or in the online appendix.

[4]See also Sgroi (2002) and Gunay (2008a, b). A textbook treatment of social learning and delay can be found in Chamley (2004).

[5]See, for example, Bose *et al.* (2006, 2008), Aoyagi (2010), and Bhalla (2013).

examined here is the main theme of the literature on durable good monopoly that begins with the Coase conjecture. The subject is also extensively studied in the marketing literature on strategic consumers.[6] The possibility that the buyers face uncertainty in their valuations is considered, among others, by Yu *et al.* (2015), and Bhalla (2012).[7] Yu *et al.* (2015) study a two-period model of monopolistic sales when consumers learn about their valuations in the second period and the monopolist can control the number of products sold in each period. Bhalla (2012) studies a two-period model of monopolistic sales in which two consumers each observe a noisy signal about the binary product quality. When only consumer 1 is active in period 1 and may delay his decision until period 2, Bhalla (2012) shows that equilibrium pricing depends on the prior probability of the high quality product. Unlike in these models, the timing and content of information about the valuations in the present model is endogenously determined by the consumers' equilibrium behavior.

Problems in which firms with differentiated products compete in price for consumers who may delay their decisions are studied by Chen and Zhang (2009), Levin *et al.* (2009), and Liu and Zhang (2013). In Chen and Zhang (2009), the market consists of two segments that are loyal to either firm, and one segment that is opportunistic. Levin *et al.* (2009) also suppose that the market consists of multiple segments and that the valuation of each product is randomly determined every period. Liu and Zhang (2013) formulate a model of vertical product differentiation when consumer valuations are random but fixed over the periods.[8]

In summary, to the best of our knowledge, the literature on the price competition by durable good sellers has studied problems in which consumers know their valuations from the outset or learn about those over time through an exogenous channel. On the other hand, the literature on social learning with strategic pricing has only looked at the problems in which consumers make a single purshase decision.[9] Our model hence marks a departure from the literature with the combination of the following elements:

[6]Beginning with Besanko and Winston (1990), one central question in this literature is what happens to the seller's revenue when the consumers become non-myopic and can delay their decisions. See Gönsch *et al.* (2012) for an extensive survey of the literature.

[7]Gunay (2014) considers a model in which the seller but not the buyers is privately informed of the quality of its good.

[8]Mak *et al.* (2012) consider price competition when one buyer alternates between two sellers who supply identical products. Anton *et al.* (2014) study dynamic price competition against a single strategic buyer when the sellers' capacity is their strategic variable.

[9]The unique exception is Jing (2011). In Jing (2011), however, social learning is not a Bayesian process and the probability with which buyers learn their true valuations in period 2 is proportional to the number of buyers who make a purchase in period 1.

- Possible delay in buyers' purchase decisions.
- Social learning about valuations by buyers.
- Dynamic price competition by firms.

As mentioned in the Introduction, we find that the buyers' social learning incentives result in a discount in the firms' period 1 prices compared with the one-shot equilibrium level. A related observation based on a different logic is made by Caminal and Vives (1996), who show in a two-period duopoly model that the firms' period 1 prices are lower than the one-shot equilibrium level when period 2 consumers have an incentive to learn the quality of the goods from their market shares in period 1.[10]

3 Model

Two firms A and B sell durable goods A and B, respectively, over two periods $t = 1, 2$ to two buyers $i = 1, 2$. Each buyer demands at most one unit of either good, and buyer i's *intrinsic* valuations of good A and B are denoted by α_i and β_i, respectively. The intrinsic valuations are determined by buyer i's *type* s_i which is drawn from the uniform distribution over the unit interval $S_i = [0, 1]$.[11] Specifically, α_i and β_i are given by

$$\alpha_i = u + 1 - s_i \quad \text{and} \quad \beta_i = u + s_i, \tag{1}$$

for some constant $u > 0$. In this formulation, hence, i's type

$$s_i = \frac{1}{2}(\beta_i - \alpha_i + 1)$$

represents the relative superiority of good B over good A in terms of i's intrinsic valuations. Furthermore, the average valuation of the two goods is constant: $\frac{1}{2}(\alpha_i + \beta_i) = u + \frac{1}{2}$.[12] As mentioned in the Introduction, each buyer is concerned about the *extrinsic*

[10] In an extension, Caminal and Vives (1999) study price dynamics over a longer horizon.

[11] Note that s_i and s_j have independent distributions. If consumers' intrinsic valuations are noisy signals about the same aspect of the product, then it will be more natural to suppose that they are positively correlated with each other. On the other hand, if those signals are about different aspects of the product (*e.g.,* driving performance and fuel efficiency of an automobile), they can be independent as formulated here. See Section 11 for more discussion on this assumption. The independent-signal interdependent-value formulation is often used in the mechanism design literature. See for example Jehiel and Moldovanu (2001). See also Aoyagi (2010).

[12] This is assumed in order to reduce the dimension of the private information. Alternative specifications are possible. For example, one can think of firm A as selling a benchmark good whose value is known with certainty, and there is uncertainty only in the value of good B. The present specification helps us keep symmetry between the two firms.

rather than intrinsic valuation of the good he purchases. Suppose that a buyer's extrinsic valuation of each good equals the weighted average of his intrinsic valuation and the other buyer's extrinsic valuation of the same good.[13] In other words, if we let v_i and w_i denote buyer i's extrinsic valuations of goods A and B, respectively, then

$$v_i = (1-\lambda)\,\alpha_i + \lambda v_j \quad \text{and} \quad w_i = (1-\lambda)\,\beta_i + \lambda w_j. \tag{2}$$

where $\lambda \in [0,1)$ is a constant expressing the degree of dependence of i's extrinsic valuation on the other buyer's valuation. Using (1) and (2) and the corresponding expressions for j, we can express v_i and w_i as a function of s_i and s_j:

$$v_i = u + (1-k)(1-s_i) + k(1-s_j),$$

and

$$w_i = u + (1-k)s_i + ks_j,$$

where $k = \frac{\lambda}{1+\lambda} \in [0, \frac{1}{2})$. Our analysis in what follows is built on these expressions of v_i and w_i.[14] Assume that u satisfies $u > \frac{1}{2} - k$.[15] When $k > 0$, the two buyers' valuations of the goods are interdependent, and the larger is k, the more dependent buyers are on the other buyer's type. Since $k < 1/2$, each buyer places more weight on his own type than the other buyer's type. On the other hand, when $k = 0$, the valuations are independent.

The game proceeds as follows: In period 1, the two firms publicly and simultaneously quote prices p_A^1 and p_B^1 of their own goods. The two buyers then make simultaneous decisions on whether to buy either good or not buy and wait. If a buyer chooses to buy either good, then the decision is irreversible and he makes no further decision. The buyers' decisions in period 1 are publicly observed. If there is at least one buyer who chooses to wait in period 1, the two firms again publicly and simultaneously quote prices p_A^2 and p_B^2 in period 2. Any buyer still in the market in period 2 then chooses to buy either good or not buy. Let $\delta \in (0,1)$ denote the common discount factor of the firms and buyers. For the buyers, this means that when they buy either good in period 2, the

[13]This corresponds to the conspicuous consumption interpretation (Footnote 2). If the extrinsic valuation is instead the weighted average of the two instrinsic valuations, then k introduced below equals λ.

[14]As a possible alternative, one may formulate the valuation functions so that the own type is not multiplied by $1-k$. We have found that none of the qualitative conclusions in this paper are affected under this alternative specification. The present formulation avoids confounding the effects of higher interdependence and higher valuations associated with a higher value of k.

[15]This ensures that the buyers' participation constraint does not bind in the period 2 price equilibrium. Note also that the range of extrinsic valuations is constant regardless of the value of k.

value of the good as well as its price is discounted by δ. For example, when buyer i buys A in period 1 for price p_A^1, his payoff equals $v_i - p_A^1$, but when he buys it in period 2 for price p_A^2, his payoff equals $\delta(v_i - p_A^2)$.[16]

Each firm f chooses its price p_f^t in period t from the set $\boldsymbol{R}_+$ of non-negative real numbers, whereas each buyer i makes his choice d_i^t in period t from the set $D = \{A, B, \emptyset\}$, where $d_i^t = \emptyset$ represents i's decision to make *no purchase* in period t. Any buyer i who chooses to buy neither good in period 1 makes another decision in period 2 so that $d_i^1 = \emptyset$ can alternatively be interpreted as the decision to *wait*. A period 1 *history* $h = (p^1, d^1)$ then consists of a pair $p^1 = (p_A^1, p_B^1) \in \boldsymbol{R}_+^2$ of the prices quoted by the two firms as well as a pair $d^1 = (d_1^1, d_2^1)$ of the decisions of the two buyers. Denote by $H = \boldsymbol{R}_+^2 \times D^2$ the set of all period 1 histories. For $i = 1$, 2, let

$$H_i = \left\{h = (p^1, d^1) \in H : d_i^1 = \emptyset\right\}$$

be the set of period 1 histories along which buyer i waits, and

$$H_{12} = H_1 \cup H_2$$

be the set of histories along which at least one buyer waits. Firm f's strategy consists of its price σ_f^1 in period 1 as well as the mapping $\sigma_f^2 : H \to \boldsymbol{R}_+$ that determines its period 2 price $p_f^2 = \sigma_f^2(h)$ as a function of the period 1 history $h \in H$. On the other hand, buyer i's strategy is a mapping $\tau_i^1 : S_i \times \boldsymbol{R}_+^2 \to D$ that determines his period 1 choice as a function of his type s_i and the period 1 prices p^1, along with a mapping $\tau_i^2 : S_i \times \boldsymbol{R}_+^2 \times H \to D$ that determines his period 2 choice as a function of his type s_i, the period 1 history h as well as the period 2 price pair p^2. Since buyer i has a decision to make in period 2 only if he chooses to wait in period 1, we impose the restriction that $\tau_i^2(s_i, p^2, h) = \emptyset$ if $h \notin H_i$. Each firm f has zero marginal cost of production and its profit simply equals the sum of prices at which the buyers buy its product.[17]

We will consider a perfect Bayesian equilibrium (PBE) of this game with an additional requirement that beliefs be obtained through Bayes rule from the buyers' strategies even when the period 1 price pair is off the path of play.[18] In other words, given *any* period 1 history $h = (p^1, d^1)$, where p^1 is on the path or not, we require that the conditional distribution $P(\cdot \mid h)$ be derived through Bayes rule whenever $P(\tau_i^1(s_i, p^1) = d_i^1,\, \tau_j^1(s_j, p^1) = d_j^1) > 0$. Given any such P, the triplet (σ, τ, P) is an

[16]In other words, the usage value of good A is $(1-\delta)v_i$ for period 1 and δv_i for period 2 (and beyond).

[17]Having a positive marginal cost does not change the analysis.

[18]Note that in the standard PBE, the belief is obtained through Bayes rule only along the equilibrium path. Our requirement would be implied by consistency in the definition of a sequential equilibrium which is defined for finite games.

equilibrium if the firms' strategies σ and the buyers' strategies τ are both sequentially rational.[19]

4 Sorting of Buyer Types in Period 1

We begin with the analysis of the buyers' equilibrium strategies in period 1. Intuitively, the buyer types who do not have a strong signal about the relative valuation of the two goods will attempt to wait and see the decision of the other buyer to gather more information. We make this intuition precise by showing that the buyers' equilibrium strategies in period 1 satisfy the *sorting condition*: For any price pair p^1, there exist $x(p^1)$ and $y(p^1)$ with $0 \leq x(p^1) \leq y(p^1) \leq 1$ such that

$$\tau_i^1(s_i, p^1) = \begin{cases} A & \text{if } s_i < x(p^1), \\ \emptyset & \text{if } x(p^1) < s_i < y(p^1), \\ B & \text{if } s_i > y(p^1). \end{cases} \tag{3}$$

In other words, when faced with p^1, buyer i chooses A if his type is at the lower-end of the type space, B if it is at the higher-end, and $\emptyset$ if it is in the middle.[20] For simplicity, we often omit the dependence of the thresholds on p^1 and simply write x and y.

Lemma 1. *Suppose that (σ, τ, P) is an equilibrium. For any buyer i and period 1 price profile $p^1 = (p_A^1, p_B^1)$, τ_i^1 satisfies the sorting condition (3) for some $x = x(p^1)$ and $y = y(p^1)$ such that $0 \leq x \leq y \leq 1$. Furthermore, if $0 < x < y$, then type x is indifferent between choosing A and waiting in period 1, and if $x < y < 1$, then type y is indifferent between choosing B and waiting in period 1.*

The intuition behind Lemma 1 is as follows: Suppose that there is some type s_i for whom choosing A in period 1 is optimal. Consider any type $s_i' < s_i$. First, any such s_i' will strictly prefer A to B in period 1. Second, s_i' also strictly prefers choosing A in period 1 to waiting. To see this, suppose first that type s_i would optimally choose A in period 2 regardless of history when forced to wait. Type s_i' would also choose A when forced to wait, but choosing A in period 1 is better than waiting by a larger margin because waiting is more costly to s_i' under positive discounting. If, on the other hand, type s_i would optimally choose B in period 2 after some history when forced to wait, then type s_i' again strictly prefers choosing A in period 1 because his valuation of B is lower than that of type s_i. It follows that type s_i' strictly prefers choosing A to waiting in period 1.

[19] See the appendix for the formal definitions.

[20] Note that one or more of the intervals may be empty.

5 Equilibrium in Period 2

With the specification of the buyers' behavior in period 1, we now describe an equilibrium in period 2. Consider buyer i's problem in period 2 following history $h \in H_i$ along which he chooses to wait $d_i^1 = \emptyset$ in period 1. Facing the price pair p^2 in period 2, buyer i of type s_i chooses A, B or $\emptyset$ in period 2 depending on which one of

$$E\left[v_i \mid s_i, h\right] - p_A^2, \quad E\left[w_i \mid s_i, h\right] - p_B^2, \quad \text{and} \quad 0$$

is the largest, where the conditional expectation $E\left[\cdot \mid h\right]$ is taken with respect to $P\left(\cdot \mid h\right)$. Let $e_j(h)$ be the expected value of buyer j's type s_j implied by the period 1 history h:

$$e_j(h) = E[s_j \mid h] = E[s_j \mid p^1, d_j^1]. \tag{4}$$

Using $e_j(h)$, we can write

$$E\left[v_i \mid s_i, h\right] = u + 1 - (1-k)s_i - ke_j(h), \quad \text{and} \quad E\left[w_i \mid s_i, h\right] = u + (1-k)s_i + ke_j(h).$$

It follows that buyer i's equilibrium strategy τ_i^2 in period 2 must satisfy

$$\tau_i^2(s_i, p^2, h) = \begin{cases} A & \text{if } s_i < \min\left\{\frac{1-2ke_j(h)-p_A^2+p_B^2}{2(1-k)}, \frac{u+1-ke_j(h)-p_A^2}{1-k}\right\}, \\ B & \text{if } s_i > \max\left\{\frac{1-2ke_j(h)-p_A^2+p_B^2}{2(1-k)}, \frac{-u-ke_j(h)+p_B^2}{1-k}\right\}, \\ \emptyset & \text{if } \frac{u+1-ke_j(h)-p_A^2}{1-k} < s_i < \frac{-u-ke_j(h)+p_B^2}{1-k}. \end{cases} \tag{5}$$

Consider next the firms' game in period 2 following $h \in H_i$ along which buyer i chooses to wait in period 1. It follows from (5) that the firms' period 2 payoffs from buyer i along $h \in H_i$ are given by

$$\pi_{A,i}^2(p^2 \mid \tau_i^2, h) = p_A^2 P\left(\tau_i^2(s_i, p^2, h) = A \mid h\right),$$
$$\pi_{B,i}^2(p^2 \mid \tau^2, h) = p_B^2 P\left(\tau_i^2(s_i, p^2, h) = B \mid h\right).$$

When buyer j's decision is described by (3), then $e_j(h)$ defined in (4) equals:

$$e_j(h) = \begin{cases} \frac{x}{2} & \text{if } d_j^1 = A, \\ \frac{x+y}{2} & \text{if } d_j^1 = \emptyset, \\ \frac{1+y}{2} & \text{if } d_j^1 = B. \end{cases} \tag{6}$$

The following lemma describes the equilibrium of the period 2 game when both buyers use the same period 1 strategy (3) with $x < y$, and when one or two buyers choose to delay. Note that the conditional probability $P(\cdot \mid h)$ of s_i given $h \in H_i$ is the uniform distribution over the interval (x, y).

Lemma 2. *Let p^1 be any period 1 price profile and $x = x(p^1)$ and $y(p^1) = y$ be the corresponding critical types as specified in (3). If $x < y$, then the equilibrium price profile $\left(\sigma_A^2(h), \sigma_B^2(h)\right)$ in period 2 following history $h = (p^1, d^1) \in H_{12}$ is unique and given as follows:*

a) (interior equilibrium) If $1 - 2ke_j(h) \in [2(1-k)(2x-y),\, 2(1-k)(2y-x)]$,[21] *then*

$$\begin{aligned}&\left(\sigma_A^2(h), \sigma_B^2(h)\right)\\ &= \left(\frac{1 - 2ke_j(h) + 2(1-k)(y-2x)}{3},\, \frac{-1 + 2ke_j(h) + 2(1-k)(2y-x)}{3}\right),\end{aligned} \tag{7}$$

and the two firms segment the market with firm A capturing $\left(x,\, \frac{1-2ke_j(h)}{6(1-k)} + \frac{x+y}{3}\right)$ *and firm B capturing* $\left(\frac{1-2ke_j(h)}{6(1-k)} + \frac{x+y}{3},\, y\right)$. *Their equilibrium payoffs (per buyer) are given by*[22]

$$\begin{aligned}\pi_A^{2*}(h) &= \frac{1}{y-x}\,\frac{\{1 - 2ke_j(h) + 2(1-k)(y-2x)\}^2}{18(1-k)},\\ \pi_B^{2*}(h) &= \frac{1}{y-x}\,\frac{\{-1 + 2ke_j(h) + 2(1-k)(2y-x)\}^2}{18(1-k)}.\end{aligned}$$

b) (A-monopolization equilibrium) If $1 - 2ke_j(h) > 2(1-k)(2y-x)$, *then*

$$\left(\sigma_A^2(h), \sigma_B^2(h)\right) = \left(1 - 2ke_j(h) - 2(1-k)y,\, 0\right), \tag{8}$$

and firm A monopolizes the market by capturing (x, y).

c) (B-monopolization equilibrium) If $1 - 2ke_j(h) < 2(1-k)(2x-y)$, *then*

$$\left(\sigma_A^2(h), \sigma_B^2(h)\right) = \left(0,\, -1 + 2ke_j(h) + 2(1-k)x\right), \tag{9}$$

and firm B monopolizes the market by capturing (x, y).

As seen, which one of the three types of the equilibria takes place in period 2 depends on the lower and upper bounds of the active buyer types (x and y) in that period as well as the expected type e_j of the other buyer.[23] In either monopolization equilibrium, the

[21] Since $y \geq x$, $2(1-k)(2x-y) \leq 2(1-k)(2y-x)$.

[22] $\pi_f^{2*}(h) = \pi_f^2(\sigma^2(h) \mid \tau^2, h)$ is firm f's (per buyer) payoff in period 2 along the history $h = (p^1, d^1)$ when the equilibrium strategies σ^2 and τ^2 are played in period 2. Given the symmetry between the buyers, firm f's (per buyer) payoff from both buyers equals that from a single buyer i.

[23] Figures 6 and 7 in the online Appendix illustrate the best-response correspondences and the equilibrium price profile.

monopolizing firm drives the price of the other firm down to zero, and the extreme type who prefers the other good the most in the market (*i.e.,* type x in the B-monopolization equilibrium and type y in the A-monopolization equilibrium) is made indifferent between the two goods. For example, in the B-monopolization equilibrium,

$$\begin{aligned} &\text{type } x\text{'s payoff from choosing } A \\ &= u + 1 - (1-k)x - ke_j(h) - 0 \\ &= u + (1-k)x + ke_j(h) - (-1 + 2ke_j(h) + 2(1-k)x) \\ &= \text{type } x\text{'s payoff from choosing } B. \end{aligned} \tag{10}$$

The same holds for type y in the A-monopolization equilibrium. This observation turns out critical for the equilibrium price dynamics as seen in the next section.

6 Equilibrium Price Dynamics

In this section, we make a critical observation on the relationship between the period 1 price and the period 2 prices in equilibrium. Fix any price pair $p^1 = (p^1_A, p^1_B)$ on or off the equilibrium path in period 1, and let $x = x(p^1)$ and $y = y(p^1)$ be the two critical buyer types as described in Lemma 1. While the firms price offers in period 2 vary with the buyers' decisions in period 1, we show that as long as $0 < x < y$, the ex ante expected payoff of the type x buyer is the same whether he chooses A in period 1 or when he waits and chooses A in period 2 after any decision by the other buyer. Likewise, when $x < y < 1$, the ex ante expected payoff of the type y buyer is the same whether he chooses B in period 1 or when he waits and chooses B in period 2 after any decision by the other buyer. Analysis of the equilibrium in the subsequent sections fully exploits this indifference property of the critical types.

Lemma 3. *(Indifference property) Suppose that (σ, τ, P) is an equilibrium. Let $p^1 = (p^1_A, p^1_B)$ be any period 1 price pair, and $x = x(p^1)$ and $y = y(p^1)$ be the corresponding critical types. If $0 < x < y$, then buyer i of type x is indifferent between choosing A in period 1 and waiting and then choosing A after any decision d^1_j by buyer j in period 1. Likewise, if $x < y < 1$, then buyer i of type y is indifferent between choosing B in period 1 and waiting and then choosing B in period 2 after any decision d^1_j of buyer j in period 1. That is, for any $h = (p^1, d^1) \in H_i$,*

$$\begin{aligned} 0 < x < y \quad &\Rightarrow \quad E\left[v_i - p^1_A \mid s_i = x\right] = \delta E\left[v_i - \sigma^2_A(h) \mid s_i = x\right], \\ x < y < 1 \quad &\Rightarrow \quad E\left[w_i - p^1_B \mid s_i = y\right] = \delta E\left[w_i - \sigma^2_B(h) \mid s_i = y\right]. \end{aligned}$$

The intuition behind Lemma 3 is as follows:[24] Consider buyer i of the critical type x. Suppose for example that when buyer i waits, we have in period 2 an A-monopolization equilibrium if buyer j chooses A in period 1, an interior equilibrium if j chooses to wait, and a B-monopolization equilibrium if j chooses B. Since type x is at the lower end of the interval in the period 2 market, he will choose A in the interior equilibria as well as in the A-monopolization equilibrium. Suppose then that j chooses B so that the B-monopolization equilibrium is played in period 2. As seen in (10), type x (who prefers A the most in the market) is just indifferent between A and B in this equilibrium, and hence A is also an optimal choice for (only) type x in this equilibrium. It follows that regardless of the other buyer's move in period 1, the unconditional choice of A in period 2 is optimal for type x. Since type x is indifferent between "choosing A in period 1" and "waiting and making an optimal contingent choice in period 2," he is also indifferent between "choosing A in period 1" and "waiting and making a non-contingent choice of A in period 2". This argument does not depend on which equilibrium is played in period 2, and whether the period 1 price pair is on the equilibrium path or not. Furthermore, Lemma 3 does not depend on our assumption that the type distribution is uniform.

As an immediate consequence of the indifference property, we have the super-martingale property of the price dynamics as follows.

Corollary 4. *(Super-martingale property) Under the conditions of Lemma 3, the price path is a super-martingale in the sense that the expected price in period 2 is lower than the period 1 price both on and off the equilibrium path. That is, for any period 1 price pair* $p^1 = (p_A^1, p_B^1)$ *and* $h = (p^1, d^1) \in H_{12}$*, we have*

$$p_A^1 > E[\sigma_A^2(h)] \quad \text{and} \quad p_B^1 > E[\sigma_B^2(h)].$$

Equilibrium price dynamics is one central topic in the literature on dynamic sales. Bose *et al.* (2008) and Bhalla (2013) both show in their respective sequential sales models that the price path is a super-martingale in the sense that the *ex ante* expected prices go down with the progress of sales. On the other hand, in a two-period model in which only one consumer arrives in period 1 and may delay, Bhalla (2012) shows that the prices can either increase or decrease over periods depending on the prior belief about the quality

[24] The indifference conditions can also be written as: $p_A^1 - \delta E\left[\sigma_A^2(h)\right] = (1-\delta)\, E\left[v_i \mid s_i = x\right]$ and $p_B^1 - \delta E\left[\sigma_B^2(h)\right] = (1-\delta)\, E\left[w_i \mid s_i = y\right]$. In other words, Lemma 3 states that the difference between the period 1 price and the discounted expected period 2 price equals the depreciation in the expected value (or the period 1 usage value) of the good for the critical type.

of the good. In a model of online sales with random arrival of consumers, Gallien (2006) shows that the price path is a sub-martingale.

Although our main analysis concerns the case of positive discounting, the indifference property established above is continuous at the limit $\delta \to 1$. In fact, as $\delta \to 1$, the indifference of types x and y reduces to

$$p_A^1 = E\left[\sigma_A^2(h)\right] \quad \text{and} \quad p_B^1 = E\left[\sigma_B^2(h)\right]. \tag{11}$$

In other words, the expected price in period 2 equals the period 1 price, and hence the price is a martingale in this limiting case.[25]

7 Impossibility of a Preemptive Equilibrium

We now proceed to the analysis of the equilibrium. In this section, we examine whether or not the equilibrium can be *preemptive* in the sense that the firm which successfully attracts one buyer in period 1 also attracts any remaining buyer in period 2. In other words, along the equilibrium path, the choice of A by a single buyer in period 1 is followed by the A-monopolization equilibrium in period 2, and the choice of B is followed by the B-monopolization equilibrium in period 2.[26]

Suppose that (σ, τ, P) is a symmetric preemptive equilibrium. Let $x = x(\sigma^1)$ and $y = y(\sigma^1)$ be the critical types in (3) under the equilibrium price profile σ^1 in period 1. By symmetry, we have $y = 1 - x$.

First, by Lemma 2, the A-monopolization equilibrium is played after $h = (\sigma^1, d^1)$ with $d^1 = (\emptyset, A)$ if and only if

$$1 - 2ke_j(h) \geq 2(1-k)(2y - x) \quad \Leftrightarrow \quad 1 - 2k \cdot \frac{x}{2} \geq 2(1-k)\left\{2(1-x) - x\right\}.$$

This along with $y = 1 - x \geq x$ implies that the relevant range of x is given by

$$x \in \left[\frac{3-4k}{6-7k}, \frac{1}{2}\right]. \tag{12}$$

By symmetry, this condition is also necessary and sufficient for the B-monopolization equilibrium to be played after the choice of B in period 1 by the other buyer. Furthermore, the interior equilibrium with $e_j(h) = \frac{1}{2}$ follows $d^1 = (\emptyset, \emptyset)$.

[25] Weber (1981) shows that the price path is a martingale in a model of sequential auctions.

[26] As will be seen in Proposition 9, such a property characterizes buyer behavior when the firms engage in marginal cost pricing in both periods.

Next, by the indifference property (Lemma 3), type x is indifferent between choosing A in period 1 and waiting and then choosing A in period 2 after every history. Substituting the equilibrium prices in period 2 from Lemma 2, we can express this condition as

$$\begin{aligned}\sigma_A^1 &= (1-\delta)\left\{u+1-\frac{k}{2}-(1-k)x\right\}+\delta E[\sigma_A^2(h)] \\ &= (1-\delta)\left\{u+1-\frac{k}{2}-(1-k)x\right\}+\delta\left\{(6-7k)x^2-(5-6k)x+1-k\right\}.\end{aligned} \tag{13}$$

When x satisfies (12), we can verify that

$$\frac{k(1-k)}{6-7k} \leq (6-7k)x^2-(5-6k)x+1-k \leq \frac{k}{4}. \tag{14}$$

(13) and (14) together imply that when δ is close to one, the period 1 price σ_A^1 in a preemptive equilibrium, if any, must be fairly low.[27] The following proposition shows that this generates an incentive for the firms to deviate and increase its price. In fact, setting a sufficiently high price in period 1 is a profitable deviation from σ_f^1 although it implies giving up the market share entirely in period 1: The deviating firm can in some cases monopolize the period 2 market.

Proposition 5. *(Impossibility of a Preemptive Equilibrium) For δ sufficiently close to one, there exists no symmetric equilibrium (σ, τ, P) in which the buyer types who wait in period 1 always choose the same firm as the other buyer who makes a purchase in period 1.*

This impossibility result may as well depend on the uniform distribution assumption. However, what is behind this result is the general observation that the buyer decisions are influenced by their expectation of the change in the firms' pricing behavior in period 2 in response to period 1 histories. More specifically, under the assumption of preemption, the corner equilibrium that follows a single purchase in period 1 implies fierce competition in period 2 over the small set of active buyer types who are concentrated around 1/2. The low expected price in period 2 then leads to the low period 1 price through the indifference property. This induces the firms to deviate and increase their price in period 1.

8 Existence of a Non-Preemptive Equilibrium

Having seen in the previous section that the equilibrium cannot be preemptive, we turn to the alternative possibility where the period 2 equilibrium is always in the interior.

[27] For example, it is significantly lower than $1-k$, the equilibrium price in the one-period model.

In other words, even if only one firm wins a buyer in period 1, some buyer types still choose the other firm in period 2. In this section, we present the main theorem of the paper that proves the existence of such an equilibrium.

We begin our analysis with the consideration of the one-period model in which the firms quote prices once and the buyers make a single purchase decision. Since such a game is equivalent to the period 2 game with $x = 0$, $y = 1$ and $e_j(h) = 1/2$, we can use Lemma 2 to characterize its equilibrium as follows.

Proposition 6. *In the one-period game, the equilibrium price profile is unique and given by* $(p_A, p_B) = (1-k, 1-k)$*, and the firms segment the market with firm A capturing* $[0, 1/2)$ *and firm B capturing* $(1/2, 1]$.

Proposition 6 demonstrates the immediate consequence of higher interdependence between the buyers. The higher is the parameter k, the more similar are the buyers' preferences and the more intense is the competition between the firms.[28] Now define the period 1 prices adjusted by the interdependence parameter:

$$q_A = \frac{p_A^1}{1-k}, \quad \text{and} \quad q_B = \frac{p_B^1}{1-k}. \tag{15}$$

It then follows from Proposition 6 that regardless of k, the adjusted prices are unity in the equilibrium of the one-period game:

$$(q_A, q_B) = (1, 1).$$

As will be seen, this observation is useful for the interpretation of the equilibrium of the original model.

Recall now that H_i is the set of histories along which buyer i waits in period 1. For any $h \in H_i$, indifference of type x between "choosing A in period 1" and "waiting and then choosing A" (Lemma 3) can be explicitly written as:

$$u + 1 - (1-k)x - \frac{k}{2} - p_A^1 = \delta E\left[u + 1 - (1-k)x - ke_j(h) - \sigma_A^2(h)\right]. \tag{16}$$

Likewise, indifference of type y can be explicitly written as:

$$u + (1-k)y + \frac{k}{2} - p_B^1 = \delta E\left[u + (1-k)y + ke_j(h) - \sigma_B^2(h)\right]. \tag{17}$$

Suppose now that in the neighborhood of some period 1 price pair p^1, every history h is followed by an interior equilibrium in period 2. This would be true if the critical

[28] If we adopted an alternative specification of the valuation function as described in footnote 14, then the equilibrium price of the one-shot model would be 1. In this case, the intensity of competition associated with a higher k is offset by the increase in the valuations.

types x and y in (3) for any price pair in the neighborhood of p^1 satisfy the conditions of Lemma 2(a). For x and y satisfying these conditions, the expected period 2 price of each good (*i.e.*, $E[\sigma_A^2(p^1, d_i^1 = \emptyset, d_j^1)]$ and $E[\sigma_B^2(p^1, d_i^1 = \emptyset, d_j^1)]$) can be computed again using Lemma 2(a) and expressed as a function of x and y. Substituting these expected prices into the indifference conditions (16) and (17), we can express the period 1 adjusted prices q_A and q_B in terms of x and y. These equations are then solved to express x and y in terms of q_A and q_B. Under the symmetric equilibrium price profile $(q_A, q_B) = (q^*, q^*)$, these expressions reduce to:

$$x = 1 - y = \frac{\delta}{3-\delta} + \mu - \frac{q^*}{1+\delta}. \tag{18}$$

We use the expression of x and y in terms of (q_A, q_B) to write firm A's overall profit Π_A as a function of the adjusted prices: $\hat{\Pi}_A(q_A, q_B)$. The first-order condition for the symmetric equilibrium price $q = q^*$ is given by $\frac{\partial \hat{\Pi}_A}{\partial q_A}(q, q) = 0$, and as seen in the Appendix, a non-negative solution to this equation is given by:

$$q^* = \begin{cases} \frac{-C + \sqrt{C^2 + 3\delta k^2 D}}{3\delta k^2} & \text{if } k > 0, \\ \frac{D}{2C} & \text{if } k = 0, \end{cases} \tag{19}$$

where

$$\begin{aligned} C &= (1+\delta)\left[18(1-k)^2 - \frac{\delta k^2}{3-\delta}\left\{-6\nu + 3(1-\delta)\mu + \frac{\delta(3-5\delta)}{3-\delta}\right\}\right], \\ D &= 18(1-k)^2(1+\delta)^3\left[\mu + \frac{\delta}{3-\delta} + \frac{4\delta}{3(1+\delta)(3-\delta)}\left\{2\lambda + \frac{3(1-\delta)}{3-\delta}\right\}\right] \\ &\quad + \delta k^2 (1+\delta)^2 \left[\frac{1+\mu-\nu}{3-\delta}\left\{(3+\delta)\nu - 2\delta(1-\mu) + \frac{3\delta(1+\delta)}{3-\delta}\right\}\right. \\ &\qquad \left. - \left(\nu + \frac{\delta}{3-\delta}\right)\left(1 - \mu - \frac{\delta}{3-\delta}\right)\right], \end{aligned}$$

and

$$\begin{aligned} \mu &= \frac{1-\delta}{(1+\delta)(3-\delta)(1-k)}\left\{(3+\delta)\left(u + 1 - \frac{k}{2}\right) - 2\delta\left(u + \frac{k}{2}\right)\right\}, \\ \nu &= \frac{1-\delta}{(1+\delta)(3-\delta)(1-k)}\left\{2\delta\left(u + 1 - \frac{k}{2}\right) - (3+\delta)\left(u + \frac{k}{2}\right)\right\}, \\ \lambda &= \nu - 2\mu. \end{aligned} \tag{20}$$

Theorem 7. *(Non-preemptive equilibrium) Let q^* be given by (19). If δ is sufficiently close to one, there exists a symmetric equilibrium (σ, τ, P) in which the firms quote $\sigma_A^1 = \sigma_B^1 = (1-k)q^*$ in period 1 and the critical buyer types x and y in period 1 are given by (18).*

The proof in the Appendix constructs the equilibrium by specifying the buyer response to every off-equilibrium price pair in period 1.[29] For a period 1 price pair that corresponds to a unilateral deviation, this construction determines the profitability of the deviation. For illustration, suppose that firm A unilaterally deviates and slightly cuts its price in period 1. This deviation is followed by higher values of the thresholds x and y: More buyer types immediately choose A, and less buyer types immediately choose B. These thresholds then determine the active buyer types in the period 2 market and the payoff of the deviating firm there. Hence, the profitability of the price cut in period 1 depends on the change in immediate sales in period 1 as well as on the change in the payoff in period 2, both of which are caused by the change in the thresholds x and y. Evaluation of the profitability of a deviation hence requires the exact identification of the thresholds based on the indifference property.

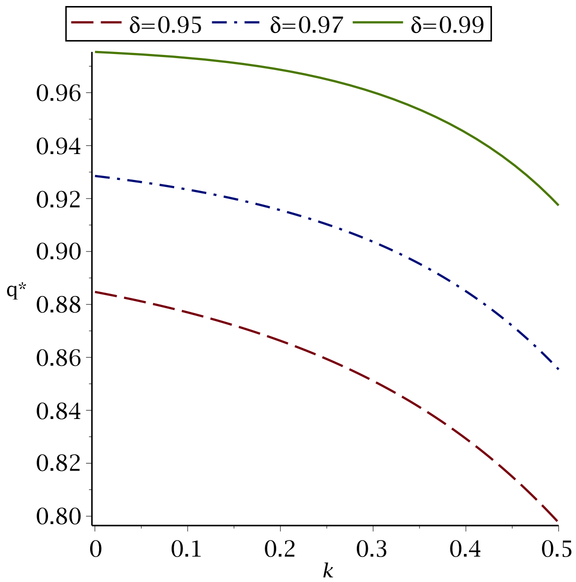

Figure 1: q^* as a function of k.

Given that the equilibrium price in the one period model equals $1 - k$ as seen in Proposition 6, we can interpret q^* as a discount in response to the increased bargaining power of the buyers with an option to wait until period 2. As can be readily verified from (19) and as illustrated in Figure 1, the adjusted price

$$\frac{\sigma_f^1}{1-k} = q^*$$

[29] We can verify that Theorem 7 holds for $\delta \geq 0.95$.

is decreasing in the interdependence parameter k for δ close to one.[30] We can interpret this as the firms' response to the stronger incentive of the more interdependent consumers to delay their decisions. A closer inspection reveals the source of this negative relationship between q^* and k: The firms' pricing game in (q_A, q_B) is shown to have strategic complementarities, and hence the equilibrium price q^* decreases with k if the marginal payoff of each firm is decreasing in k.[31] We can verify that k does not affect the marginal payoff in period 1 or that in period 2 corresponding to the buyers' intrinsic valuations. On the other hand, in the neighborhood of the equilibrium, an increase in k negatively impacts the marginal payoff in period 2 corresponding to the interdependent component of the buyers' valuations.

On the other hand, Figure 2 shows that q^* is an increasing function of δ, implying that period 1 pricing responds more strongly to the patience of the firms than that of the buyers.

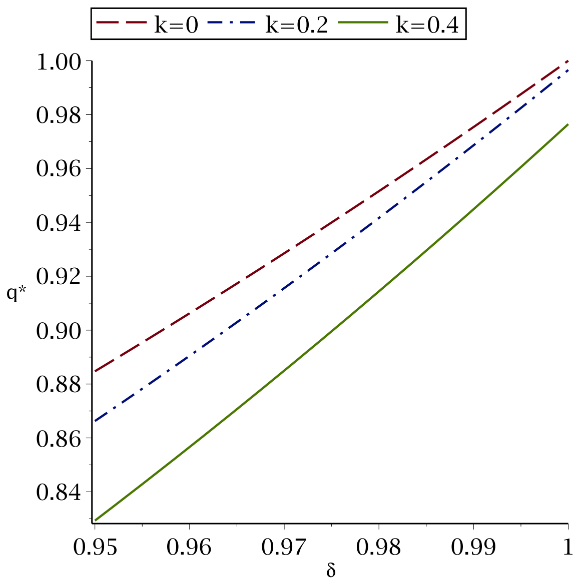

Figure 2: q^* as a function of δ.

[30]It is straightforward to verify that $\frac{\partial q^*}{\partial k} < 0$ when $\delta = 1$. The corresponding inequality for δ close to one then follows from the continuity of the derivative with respect to k.

[31]See Topkis (1998). To be precise, the argument below is pertinent to $\frac{\hat{\Pi}_A}{1-k}$ for $\delta = 1$. It however implies $\frac{\partial}{\partial k}\left(\frac{\partial \hat{\Pi}_A}{\partial q_A}\right) < 0$ for δ sufficiently large.

9 Delay

According to Theorem 7 and (18), the probability of types who wait in period 1 is given by

$$y - x = 1 - 2x = -\frac{(1-\delta)(2u+1)}{(1+\delta)(1-k)} + \frac{2q^*}{1+\delta}.$$

We can see from this that there is substantial delay when the buyers are completely independent ($k = 0$). In fact, when $k = 0$, $y - x \to 1$ in the limit as $\delta \to 1$ so that the equilibrium involves full delay.[32] More generally, for δ sufficiently close to one, q^* is decreasing in k, and so is $y - x$. In other words, there will be less delay when the buyers are more interdependent as summarized in the following corollary.

Corollary 8. *(Delay as a function of k) For δ sufficiently close to one, the probability of delay by either buyer in the equilibrium of Theorem 7 decreases as they become more interdependent.*

Corollary 8 appears counter-intuitive since in general, a more interdependent buyer is likely to have a stronger incentive to learn from the behavior of the other buyer. At first sight, it may seem that less delay follows directly from the lower period 1 price for a larger k. This intuition, however, is misleading. Consider for example the limiting case as $\delta \to 1$. In this case, as seen in Section 6, any reduction in the period 1 price is accompanied by the reduction in the expected period 2 price by the same margin.

In order to understand the nature of the relationship between the degree of interdependence and the probability of delay, it is useful to consider an alternative model in which the firms' prices are fixed and set equal to the marginal cost in both periods. If we define

$$x^0 = \frac{\delta(2-3k) - (1-k) + \sqrt{\{\delta(2-3k) - (1-k)\}^2 + 4\delta(2-3k)\overline{\Delta}}}{2\delta(2-3k)}, \tag{21}$$

[32] When $k = 0$, there exists another equilibrium with no delay as follows: The firms quote $\sigma^1 = (1, 1)$ in period 1, and all buyer types move in period 1: Type s_i chooses A if $s_i < \frac{1}{2}$ and B if $s_i > \frac{1}{2}$. The conditional distribution $P(\cdot \mid h)$ when either buyer waits (*i.e.,* after any $h \in H_{12}$) is the same as the prior (*i.e.,* the uniform distribution over $[0, 1]$). Since the period 2 equilibrium price pair along any such history is again $(1, 1)$, and since the buyers have no incentive to learn from the behavior of the other, their decision in period 1 not to wait is optimal. When $k > 0$, however, there is no equilibrium of this type. If every buyer type moves in period 1 and if the price in period 2 is the same as that in period 1, then there exists a buyer type who has an incentive to wait and see provided that they are sufficiently patient. In other words, only the first equilibrium for $k = 0$ is robust to a small perturbation in the value of k.

where

$$\overline{\Delta} = (1-\delta)\left(u + 1 - \frac{k}{2}\right) > 0, \tag{22}$$

then the buyer behavior is described as follows.

Proposition 9. *(Buyer behavior under marginal cost pricing) Suppose that the prices are fixed at marginal cost in both periods: $p_A^1 = p_B^1 = 0$ and $p_A^2 = p_B^2 = 0$. For δ sufficiently close to one, the period 1 thresholds x and y are given by $x = 1 - y = x^0$ in (21). Furthermore, if only one buyer j makes a purchase in period 1, then the other buyer i always chooses the same good as j in period 2. Furthermore, the probability of delay $1 - 2x$ is increasing in k.*

Unlike in Corollary 8, the more interdependent buyers delay more often in this alternative environment. The buyers' incentives are clear in this model: Since the prices are fixed, delay is caused purely by informational concerns. As a result, if a buyer places more weight on the other buyer's type, he has a stronger incentive to wait and see. In contrast, when the prices are endogenously determined, the buyer decision in period 1 is determined by the exact tradeoff between the informational advantage of waiting, and the increase in the period 2 prices caused by the waiting decision of more buyer types. The indifference property summarizes this tradeoff. Specifically, when the period 2 equilibria are all interior, the indifference property requires x and q^* to be inversely related by (18). Hence, if q^* decreases as a result of the increase in k as seen in Section 8, it leads to an increase in x, or equivalently, smaller delay. Table 1 summarizes some key observation about the delay probabilities.[33]

Model	$k = 0,\ \delta \to 1$	as $k \uparrow$	$k = \frac{1}{2},\ \delta \to 1$
Equilibrium	1	$\downarrow$	0.9499
MC pricing	0	$\uparrow$	1
Optimal learning	$\frac{1}{3}$	$\uparrow$	$\frac{1}{2}$

Table 1: Delay probabilities ($y - x = 1 - 2x$)

10 Social Efficiency

We next turn to the social efficiency of the buyer decisions in the equilibrium identified in Theorem 7. As mentioned in the Introduction, it is possible to consider two sources

[33]See Section 10 for the optimal learning benchmark.

of the inefficiency of the equilibrium decisions. First, a buyer's decision in period 1 may be distorted by their expectation of the firms' response in period 2 to their decisions. Second, a self-interested buyer fails to internalize the informational externalities that his own decision may inflict on the other buyer. We attempt to quantify these two different effects using some hypothetical benchmarks. For simplicity, our analysis in this section focuses on the limiting case as $\delta \to 1$.

First, consider the full-information benchmark in which the two buyers truthfully share private information about their types. In this case,

$$\text{buyer } i \text{ should choose} \begin{cases} A & \text{if } v_i > w_i \Leftrightarrow (1-k)s_i + ks_j < \frac{1}{2}, \\ B & \text{if } v_i < w_i \Leftrightarrow (1-k)s_i + ks_j > \frac{1}{2}. \end{cases}$$

It follows that the expected value of the *ex post* optimal decision is given by[34]

$$E\left[\max\left\{v_i,\, w_i\right\} - u\right] = E\left[\max\left\{1 - (1-k)s_i - ks_j,\, (1-k)s_i + ks_j\right\}\right] = \frac{3}{4}. \quad (23)$$

If, on the other hand, each buyer is informationally isolated, then the efficiency level equals

$$\frac{3-k}{4}.$$

Note that this no-information benchmark corresponds to the buyer decisions when there is either full or no delay since no informational interaction exists between the buyers in these cases.

The third benchmark is the marginal cost pricing model studied in Section 9. In this model, the price effect discussed above is absent, but the externality effect is still present since learning is self-interested. By letting $\delta \to 1$ in (21), we can see that the critical types in the limit are given by

$$x^0 = 1 - y^0 = \frac{1-2k}{2-3k}. \quad (24)$$

Furthermore, as seen in Proposition 9, if only one buyer moves in period 1, then the other buyer always chooses the same firm in period 2. The expected efficiency of the buyer decisions under marginal cost pricing can hence be computed as:

$$\frac{3-k}{4} + \frac{k^2(1-2k)(1-k)}{2(2-3k)^3}. \quad (25)$$

Our fourth and final benchmark is the optimal learning model that has marginal cost pricing *and* the buyer behavior that internalizes the informational externalities of their

[34]Throughout this section, efficiency is computed net of the constant term u.

decisions. Specifically, we suppose that each buyer chooses the period 1 thresholds x and y so as to maximize the sum of their payoffs. By the symmetry of the prices, we can again set $y = 1 - x$. Furthermore, if we let $\xi = \frac{1-kx}{2(1-k)}$, then buyer i should choose A in period 2 if (i) $d_j^1 = A$ and $s_i \in (x, \min\{\xi, 1-x\})$, (ii) $d_j^1 = \emptyset$ and $s_i \in (x, \frac{1}{2})$, or (iii) $d_j^1 = B$ and $s_i \in (x, \max\{1-\xi, x\})$.[35] Since $\xi < 1 - x$ if and only if $x < \frac{1-2k}{2-3k}$, the expected efficiency level can be computed as

$$\begin{cases} 2\left[\xi x + \frac{1}{2}(1-k)(1-2\xi^2)x + \frac{k}{2}(1-2\xi)x^2\right] + \frac{3-k}{4}(1-2x) & \text{if } x \le \frac{1-2k}{2-3k}, \\ \frac{3-k}{4} - \frac{1-3k}{2}x + (2-5k)x^2 - 2(1-2k)x^3 & \text{if } x > \frac{1-2k}{2-3k}. \end{cases}$$

It follows that the optimal threshold $x = x^*$ is given by

$$x^* = \begin{cases} \frac{1}{3} & \text{if } k \le \frac{1}{3}, \\ \frac{2-5k+\sqrt{1-5k+7k^2}}{6(1-2k)} & \text{if } k \in (\frac{1}{3}, \frac{1}{2}), \\ \frac{1}{4} & \text{if } k = \frac{1}{2}.^{36} \end{cases} \tag{26}$$

We are now ready to compare these benchmarks with the equilibrium buyer decisions identified in Theorem 7. In the limit as $\delta \to 1$, x in (18) approaches $\frac{1-q^*}{2}$. Furthermore, the critical buyer type that is indifferent between the two goods in period 2 equals $c = \frac{2-k(1-q^*)}{12(1-k)} + \frac{1}{3}$ when the other buyer chooses A in period 1. By symmetry, the critical buyer type equals $1 - c$ when the other buyer chooses B in period 1, and equals $\frac{1}{2}$ when the other buyer also waits. The expected efficiency of the buyer decisions in equilibrium can be computed as

$$2\left[cx + \frac{1}{2}(1-k)(1-2c^2)x + \frac{k}{2}(1-2c)x^2\right] + \frac{3-k}{4}(1-2x). \tag{27}$$

Figure 3 illustrates buyer i's choice of the good as a function of the type profile in equilibrium and under full information: The equilibrium choice of A is indicated by the shaded area, whereas the efficient choice is A to the left of the straight line $(1-k)s_i + ks_j = \frac{1}{2}$ and B to the right of it. Figure 4 illustrates the efficiency of the buyer decisions in equilibrium as well as in the benchmark models. As seen, the decision becomes less efficient as the interdependence parameter k increases. This is expected from Corollary 8 since more interdependent buyers tend to move in period 1 more often. It can be seen

[35] $s_i = \xi$ is the critical buyer type that is indifferent between the two goods in period 2 when the other buyer chooses A in period 1. If $\xi \ge 1 - x$, then it implies that every active buyer type in period 2 will choose A.

[36] Compared with the critical type x^0 in the marginal cost pricing model in (24), x^* in (26) is larger or smaller depending on the value of k: $x^* \lesseqgtr x^0 \Leftrightarrow k \lesseqgtr \frac{1}{3}$.

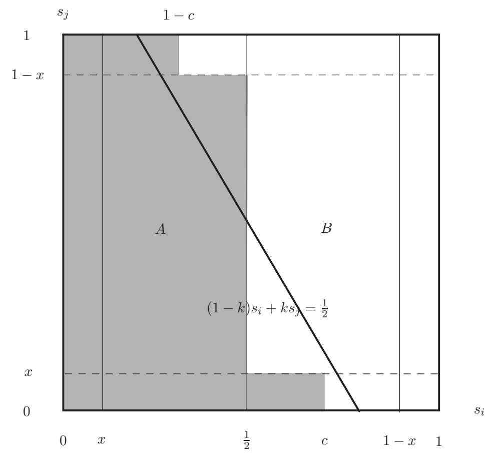

Figure 3: Efficient and equilibrium decisions by buyer i

that for small values of k, the inefficiency of equilibrium is caused almost exclusively by the price effect since marginal cost benchmark with self-interested learning yields almost the same level of efficiency as the optimal learning benchmark. On the other hand, as k grows closer to $\frac{1}{2}$, self-interest in learning appears to become a more important source of inefficiency since the marginal cost benchmark approaches the equilibrium level.[37]

11 Discussions

The assumption of the uniform distribution of the types is standard in the models of product differentiation and perhaps is the only one that admits analytical derivation of the equilibrium in our framework. While we admit that the assumption is restrictive in some ways, we also note that the specification of the distribution becomes less important when the degree of differentiation becomes small compared with the absolute values of the products as represented by the constant u in the valuation function. Furthermore, our result suggests that problems with alternative distributions can be numerically analyzed with the help of the indifference property.

Unlike in the majority of the social learning literature that assumes that a consumer's type s_i is a noisy signal of the underlying state ω, we have adopted an alternative framework in which there is no ω and the consumer types s_1 and s_2 are independent. In

[37]It is also interesting to note that the marginal cost benchmark is less efficienct than the equibrium near $k = \frac{1}{2}$. This is because at $k = \frac{1}{2}$, there is full delay (*i.e.*, $x^0 = 0$ in (24)) and hence no learning under marginal cost pricing. To the contrary, even with substantial delay, equilibrium learning is still positive at $k = \frac{1}{2}$.

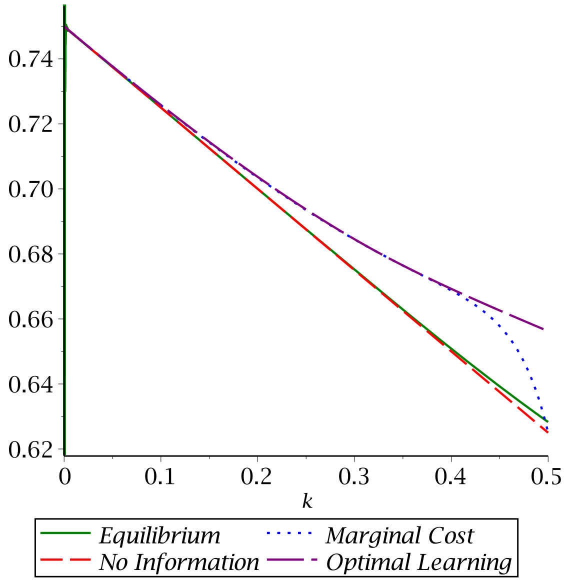

Figure 4: Efficiency as a function of k.

defense of our assumption, we should note that it eliminates a few technical problems that would arise under the alternative assumption of correlated signals about the underlying state. First, we would need to specify a family of conditional distributions of the signal for each state ω. Specification of such conditional distributions is nontrivial and any specification would involve far more complicated analysis if possible at all.[38]

Second, if the firms do not know the realization of ω, then we should consider the firms' incentive to learn ω through their pricing strategy. If they know ω, on the other hand, we should think about their signaling incentives. Our assumption helps us abstract from these considerations, which could significantly complicate the problem.

For analytical tractability, we have confined ourselves to a model with two buyers. In a market with a large number of consumers, on the other hand, a waiting consumer can observe the behavior of many other consumers, and hence may be able to form a more correct estimate about the value of the products than from observing the behavior of a single other buyer. In the correlated private signal models with the true underlying state ω as described above, a large market is often depicted as a continuous population of consumers. With the application of the law of large numbers, those models often suppose that the underlying state is perfectly revealed after the first period. The same

[38]One possibility is the binary specification of the signal. However, we have the problem of having no pure equilibrium in a stage game in this case.

cannot be said in our model, and the implications of having a continuous population in our framework are yet to be studied.

In one interesting extension, we can consider a model in which the consumers are different in their interdependence levels. Targeting a particular class of consumers is shown to be a useful sales strategy in different contexts, and it would be interesting to examine if this is also the case in the present setting.[39]

A Appendix

A.1 Payoff Functions

This section presents a formal definition of the payoff functions. For any pair $p = (p_A, p_B)$ of prices and pair $s = (s_1, s_2)$ of types, let $\pi_i(s, p, d_i)$ denote buyer i's *ex post* payoff from decision $d_i \in D$:

$$\pi_i(s,p,d_i) = \begin{cases} v_i - p_A & \text{if } d_i = A, \\ w_i - p_B & \text{if } d_i = B, \\ 0 & \text{if } d_i = \emptyset. \end{cases}$$

When the strategies of the firms and the buyers in period 2 are given, buyer i's *ex post* payoff over two periods as a function of his type as well as history $h = (p^1, d^1)$ is then written as:

$$\Pi_i(s, p^1, d^1 \mid \sigma^2, \tau^2) = \begin{cases} \pi_i(s, p^1, d_i^1) & \text{if } d_i^1 = A \text{ or } B, \\ \pi_i\left(s, \sigma^2(h), \tau_i^2(s_i, \sigma^2(h), h)\right) & \text{if } d_i^1 = \emptyset, \end{cases}$$

where $h = (p^1, d^1)$. Now for any history $h \in H$, let

$$P\left(\cdot \mid h\right)$$

denote the conditional distribution of buyer i's type s_i given h. Each firm f's period 2 payoff from buyer i is expressed as a function of the period 2 price pair p^2 as well as history h and buyer i's period 2 strategies τ_i^2:

$$\pi_{f,i}^2(p^2 \mid \tau_i^2, h) = p_f^2 P\left(\tau_i^2(s_i, p^2, h) = f \mid h\right)$$

Furthermore, when the two buyers' strategies $\tau = (\tau^1, \tau^2)$ in both periods as well as the firms' strategies $\sigma^2 = (\sigma_A^2, \sigma_B^2)$ in period 2 are given, let $\Pi_{f,i}(p^1 \mid \tau, \sigma^2)$ denote firm f's

[39] In a model where the dependence levels of consumers are observable to a monopolist seller, Aoyagi (2010) shows that it is optimal for the seller to target the least dependent consumers first and then move in the increasing order of the dependence levels.

payoff over two periods from buyer i as a function of the period 1 price pair:

$$\Pi_{f,i}(p^1 \mid \tau, \sigma^2) = p_f^1 P\left(\tau_i^1(s_i, p^1) = f\right) + E\left[\pi_{f,i}^2(\sigma^2(h) \mid \tau_i^2, h)\right],$$

where $h = (p^1, \tau_1^1(s_1, p^1), \tau_2^1(s_2, p^1))$. Firm f's *per buyer* payoffs from both buyers in period 2 and over two periods are then given by

$$\pi_f^2(p^2 \mid \tau^2, h) = \frac{1}{2}\sum_{i=1}^{2} \pi_{f,i}^2(p^2 \mid \tau_i^2, h), \quad \text{and} \quad \Pi_f(p^1 \mid \tau, \sigma^2) = \frac{1}{2}\sum_{i=1}^{2} \Pi_{f,i}(p^1 \mid \tau, \sigma^2),$$

respectively.

In period 2, for any type s_i, history $h \in H_i$, and period 2 price pair p^2, buyer i's decision $\tau_i^2(s_i, p^2, h) \in D$ in period 2 maximizes his expected utility, and for any $h \in H_{12}$ along which at least one buyer chooses to wait in period 1, the firms' price pair $\sigma^2(h)$ in period 2 is a NE of the firms' game in period 2 given the belief $P(\cdot \mid h)$ about each buyer i's type conditional on h. Formally, for each $i = 1, 2$, $h \in H_i$ and $p^2 \in \boldsymbol{R}_+^2$, τ_i^2 is sequentially rational and satisfies

$$\tau_i^2(s_i, p^2, h) \in \underset{d_i^2}{\operatorname{argmax}}\ E\left[\pi_i(s, p^2, d_i^2) \mid s_i, h\right],$$

and for each $h \in H_{12}$ and τ^2 that is sequentially rational, $\sigma^2(h)$ is sequentially rational and satisfies for $f = A$, B, and $\ell \neq f$,

$$\sigma_f^2(h) \in \underset{p_f^2}{\operatorname{argmax}}\ \pi_f^2(p_f^2, \sigma_\ell^2(h) \mid \tau^2, h).$$

Furthermore, facing any price pair p^1, buyer i's period 1 strategy τ_i^1 is sequentially rational given the sequentially rational period 2 strategies τ^2 and σ^2: For every type s_i,

$$\tau_i^1(s_i, p^1) \in \underset{d_i^1}{\operatorname{argmax}}\ E\left[\Pi_i(s_i, p^1, d_i^1 \mid \tau^2, \sigma^2)\right],$$

and the price pair $\sigma^1 = (\sigma_1^1, \sigma_2^1)$ is optimal against each other given the buyers' strategies and the firms' period 2 strategies both of which are sequentially rational: For $f = A$, B, and $\ell \neq f$,

$$\sigma_f^1 \in \underset{p_f^1}{\operatorname{argmax}}\ \Pi_f(p_f^1, \sigma_\ell^1 \mid \tau, \sigma^2).$$

Finally, the conditional distribution $P(\cdot \mid h)$ is such that for any p^1 on or off the path of play and any $d^1 = (d_i^1, d_j^1)$, if a strictly positive measure of types of buyer i choose d_i^1 when faced with p^1 (*i.e.*, $P\left(\tau_i^1(s_i, p^1) = d_i^1\right) > 0$), then $P(s_i \mid h)$ about buyer i's type s_i given $h = (p^1, d^1) \in H$ is derived through Bayes rule. $P(s_i \mid h)$ is arbitrary for any $h = (p^1, d^1)$ if buyer i's types that choose d_i^1 after p^1 have measure zero.

A.2 Proofs

The proofs of Lemma 2, Corollary 4, and Proposition 9 can be found in the online Appendix.

Proof of Lemma 1. We will show that if $\tau_i^1(s_i, p^1) = A$ for some s_i and $s_i' < s_i$, then $\tau_i(s_i', p^1) = A$. By setting $x(p^1) = \sup\left\{s_i : \tau_i^1(s_i, p^1) = A\right\}$, it would then follow that $\tau_i^1(s_i, p^1) = A$ if $s_i < x$.

Suppose that $\tau_i^1(s_i, p^1) = A$ and that $s_i' < s_i$. Since type s_i prefers choosing A to choosing B in period 1, we have

$$E[v_i \mid s_i] - p_A^1 \geq E[w_i \mid s_i] - p_B^1. \tag{28}$$

Likewise, since type s_i prefers choosing A in period 1 to waiting and then choosing either A, $\emptyset$, or B in period 2, we also have

$$E[v_i \mid s_i] - p_A^1 \geq \delta \sum_{h \in H_i(p^1)} P(h) \max\left\{E[v_i \mid s_i, h] - \sigma_A^2(h),\, 0,\, E[w_i \mid s_i, h] - \sigma_B^2(h)\right\}, \tag{29}$$

where $H_i(p^1) = \left\{h = (p^1, d^1) : d_i^1 = \emptyset\right\}$ is the set of period 1 histories along which the firms quote p^1 and buyer i chooses to wait. Note now that

$$\begin{aligned} E[v_i \mid s_i'] &= (1-k)(s_i - s_i') + E[v_i \mid s_i] > E[v_i \mid s_i], \quad \text{and} \\ E[w_i \mid s_i'] &= -(1-k)(s_i - s_i') + E[w_i \mid s_i] < E[w_i \mid s_i]. \end{aligned}$$

It then immediately follows that (28) holds for type s_i' with strict inequality so that it strictly prefers choosing A to choosing B in period 1. To see that s_i' also prefers choosing A to waiting, add $(1-k)(s_i - s_i') > 0$ to both sides of (29). We then have

$$\begin{aligned} &E[v_i \mid s_i'] - p_A^1 \\ &\geq (1-\delta)(1-k)(s_i - s_i') \\ &+ \delta \sum_{h \in H_i(p^1)} P(h) \max\Big\{E[v_i \mid s_i', h] - \sigma_A^2(h),\, (1-k)(s_i - s_i'), \\ &\qquad\qquad (1-k)(s_i - s_i') + E[w_i \mid s_i, h] - \sigma_B^2(h)\Big\} \\ &> \delta \sum_{h \in H_i(p^1)} P(h) \max\Big\{E[v_i \mid s_i', h] - \sigma_A^2(h),\, 0,\, E[w_i \mid s_i', h] - \sigma_B^2(h)\Big\}, \end{aligned}$$

which shows that (29) holds for type s_i' with strict inequality, and hence it strictly prefers choosing A to waiting. It can be shown similarly that if we define $y = \inf\{s_i :$

$\tau_i^1(s_i, p^1) = B\}$, then $\tau_i^1(s_i, p^1) = B$ for $s_i > y$. If $s_i \in (x, y)$, then we cannot have $\tau_i^1(s_i, p^1) = A$ since that would imply $\tau_i^1(s_i', p^1) = A$ for some $s_i' > x$, a contradiction. Since we cannot have $\tau_i^1(s_i, p^1) = B$ either, we must have $\tau_i^1(s_i, p^1) = \emptyset$.

Suppose that $0 < x < y$. To see that type x is indifferent between choosing A and waiting in period 1, suppose that he strictly prefers choosing A so that (29) holds with strict inequality for $s_i = x$. Since both sides of (29) are continuous in s_i, we would then have strict inequality hold for $s_i < y$ sufficiently close to x, a contradiction to the definition of x. A similar contradiction would follow if type x strictly prefers waiting to choosing A in period 1. The symmetric argument shows that type y is indifferent between choosing B and waiting in period 1 when $x < y < 1$. □

Proof of Lemma 3. We first show that if $\sigma^2(h)$ is as given by Lemma 2, then after any d_j^1, type x's payoff from *unconditionally* choosing A in period 2 equals that from following the sequentially rational strategy τ_i^2: $E\left[\pi_i^2(x, s_j, \sigma^2(h), \tau_i^2(x, h, \sigma^2(h)) \mid s_i, h\right]$, where $h = (p^1, \emptyset, d_j^1)$. For this, note that type x is the lowest type in the period 2 market. Hence, after any decision d_j^1 of buyer j, if $d^1 = (\emptyset, d_j^1)$ is followed by an interior equilibrium or an A-monopolization equilibrium (Lemma 2), then type x will optimally choose A in period 2 after d^1. On the other hand, if d^1 is followed by a B-monopolization equilibrium, then type x is just indifferent between A and B after $h = (p^1, d^1)$ as seen in (10). It follows that in period 2, choosing A unconditionally is optimal for type x regardless of buyer j's decision d_j^1 or the type of equilibrium that follows d^1. This in turn implies that type x's payoff from waiting in period 1 equals that from waiting and then unconditionally choosing A in period 2. Now in period 1, if $x > 0$ and waiting is strictly better than choosing A, then for $\epsilon > 0$ small, type $s_i = x - \epsilon > 0$ also finds it strictly better off waiting, which is a contradiction to the sequential rationality of τ_i^1. On the other hand, if $x < 1$ and choosing A in period 1 is strictly better than waiting, then for $\epsilon > 0$ small, type $s_i = x + \epsilon < 1$ finds it strictly better off choosing A in period 1, which is again a contradiction to the sequential rationality of τ_i^1. Hence, type $s_i = x$ is indifferent between choosing A and waiting in period 1. Combining the two observations together, we have

$$E\left[v_i \mid s_i = x\right] - p_A^1 = \delta E\Big[E\left[v_i \mid s_i = x, h\right] - \sigma_A^2(h) \,\Big|\, s_i = x\Big],$$

where the left-hand side is buyer i's payoff from buying A in period 1, and the right-hand side is his payoff from waiting and then unconditionally choosing A in period 2. Application of the law of iterated expectations

$$E\left[v_i \mid s_i = x\right] = E\Big[E\left[v_i \mid s_i = x, h\right] \,\Big|\, s_i = x\Big]$$

then yields the desired conclusion. The symmetric discussion proves the statement for the price of B. $\square$

Proof of Proposition 5. Firm A's payoff over two periods under σ^1 can be written in terms of x as

$$\begin{aligned}
&\Pi_A^1(\sigma)\\
&= x\sigma_A^1 + \delta(1-2x)\Big[x\pi_A^{2*}(\sigma^1,\emptyset,A) + (1-2x)\pi_A^{2*}(\sigma^1,\emptyset,\emptyset)\Big]\\
&= x(1-\delta)\left\{u + 1 - \frac{k}{2} - (1-k)x\right\} + x\delta\left\{(6-7k)x^2 - (5-6k)x + 1 - k\right\}\\
&+ \delta(1-2x)x\left\{1 - kx - 2(1-k)(1-x)\right\}\\
&+ \delta(1-2x)\frac{\{1-k+2(1-k)(1-2x)\}^2}{18(1-k)}\\
&= x(1-\delta)\left\{u + 1 - \frac{k}{2} - (1-k)x\right\} + \delta\varphi(x),
\end{aligned}$$

where

$$\varphi(x) = (-2+3k)x^3 + (5-7k)x^2 + (-3+4k)x + \frac{1-k}{2}.$$

Since φ is convex over $\left[0, \frac{5-7k}{3(2-3k)}\right]$, and since $0 < \frac{2}{5} < \frac{3-4k}{6-7k} < \frac{1}{2} < \frac{5-7k}{3(2-3k)}$, for x satisfying (12), we have

$$\varphi(x) \le \max\left\{\varphi\left(\frac{1}{2}\right), \varphi\left(\frac{2}{5}\right)\right\} = \max\left\{\frac{k}{8}, \frac{-7+43k}{250}\right\} = \frac{k}{8} < \frac{1}{16}. \qquad (30)$$

Suppose now that firm A deviates to a price p_A^1 so high that $x(p_A^1, \sigma_B^1) = 0$. Define $y = y(p_A^1, \sigma_B^1)$. There are the following three possibilities regarding the type of the period 2 equilibrium $\sigma^2(h)$ that follows $h = (p_A^1, \sigma_B^1, d_i^1 = \emptyset, d_j^1)$.

1) $\sigma^2(h)$ is an A-monopolization equilibrium for $d_j^1 = \emptyset$ and $d_j^1 = B$. $\Leftrightarrow y < \frac{1-k}{4-3k}$.

   Firm A's payoff over the two periods under (q_A, q) is given by

$$\begin{aligned}
\Pi_A^1(p_A^1, \sigma_B^1) &= \delta y\left\{1 - ky - 2(1-k)y\right\} + \delta(1-y)\left\{1 - k(1+y) - 2(1-k)y\right\}\\
&= \delta(1-k)(1-2y).
\end{aligned}$$

   Since $y < \frac{1-k}{4-3k}$, $\hat{\Pi}_A^1(q_A, q) > \delta\frac{(1-k)(2-k)}{4-3k} \ge \frac{\delta}{5}$.

2) $\sigma^2(h)$ is an A-monopolization equilibrium for $d_j^1 = \emptyset$, and an interior equilibrium for $d_j^1 = B$. $\Leftrightarrow \frac{1-k}{4-3k} \le y < \frac{1}{4-3k}$.

Firm A's payoff over the two periods under (q_A, q) satisfies

$$\Pi_A^1(p_A^1, \sigma_B^1) \geq \delta y \left\{1 - ky - 2(1-k)y\right\} = \delta y \left\{1 - (2-k)y\right\}.$$

We can readily verify that when $\frac{1-k}{4-3k} \leq y < \frac{1}{4-3k}$, the right-hand side above is $\geq \delta \frac{(1-k)(2-k^2)}{(4-3k)^2} = \delta \frac{1-k}{4-3k} \times \frac{2-k^2}{4-3k} \geq \delta \times \frac{1}{5} \times \frac{1}{2} = \frac{\delta}{10}$.

3) $\sigma^2(h)$ is an interior equilibrium for $d_j^1 = \emptyset$ and $d_j^1 = B$. $\Leftrightarrow y \geq \frac{1}{4-3k}$.

Firm A's payoff over the two periods under (q_A, q) is given by

$$\begin{aligned}\Pi_A^1(p_A^1, \sigma_B^1) &= \delta y \frac{\left\{1 - ky + 2(1-k)y\right\}^2}{18(1-k)} + \delta(1-y) \frac{\left\{1 - k(1+y) + 2(1-k)y\right\}^2}{18(1-k)} \\ &= \frac{\delta}{18(1-k)} \left[\left\{4(1-k)^2 - k^2\right\} y^2 + \left\{4(1-k)^2 + k^2\right\} y + (1-k)^2\right].\end{aligned}$$

Since $y \geq \frac{1}{4-3k} > \frac{1}{4}$, we can verify that

$$\Pi_A^1(p_A^1, \sigma_B^1) > \frac{\delta}{18(1-k)} \left[\frac{9}{4}(1-k)^2 + \frac{3}{16} k^2\right] \geq \frac{\delta}{8}.$$

By (30), for δ sufficiently close to one, we have

$$\Pi_A^1(\sigma^1) < \frac{\delta}{10} \leq \Pi_A^1(p_A^1, \sigma_B^1),$$

which implies that such a p_A^1 is a profitable deviation from σ_A^1. □

Proof of Theorem 7. Consider the following pair of a strategy profile (σ, τ) and conditional beliefs $P(\cdot \mid h)$.

– Period 1 strategies:

For q^* given in (19), the firms quote

$$\sigma_f^1 = (1-k)q^*, \tag{31}$$

and for any $(q_A, q_B) = \left(\frac{p_A^1}{1-k}, \frac{p_B^1}{1-k}\right)$, buyer i's decision is given by (3) for x and y defined as follows:

Let $\underline{q}$, $\bar{q}$ and q^0 be defined by

$$\begin{aligned}\underline{q} &= 1 - \frac{\delta(\nu - 2\mu) + 3\nu}{3} - \frac{\delta(3-4k)}{6(1-k)}, \\ \bar{q} &= \frac{1+\delta}{3-\delta} \left\{3 - 2\delta - \nu(3-\delta)\right\}, \\ q^0 &= \frac{1-\delta}{1-k}\left(u + 1 - \frac{k}{2}\right) + \frac{k\delta - 3(1-k)(1-\delta)}{4-3k}.\end{aligned} \tag{32}$$

These values are as indicated in Figure 5.[40]

1. If $(q_A, q_B) \in R_1$, *i.e.*,

$$\max\{q_A, q_B\} > \underline{q},$$

and

$$\begin{aligned} (3+\delta)q_A - 2\delta q_B &\leq (1+\delta)\{\delta + (3-\delta)\mu\}, \quad (33) \\ 2\delta q_A - (3+\delta)q_B &\geq (1+\delta)\{-3+2\delta+\nu(3-\delta)\}, \quad (34) \end{aligned}$$

then x and y are given by

$$\begin{aligned} x &= \frac{\delta}{3-\delta} + \mu - \frac{(3+\delta)\,q_A - 2\delta q_B}{(1+\delta)(3-\delta)}, \\ y &= \frac{\delta}{3-\delta} + \nu + \frac{(3+\delta)\,q_B - 2\delta q_A}{(1+\delta)(3-\delta)}. \end{aligned} \quad (35)$$

This is an expression of the indifference conditions of the critical types x and y under the assumption that every d_j^1 is followed by an interior equilibrium in period 2. More specifically, the derivation of (35) is as follows: Suppose that for any price pair in the neighborhood of p^1, we have an interior equilibrium in period 2 after any history $h \in H_i$. By Lemma 2(a), this holds if the critical types x and y in (3) satisfy

$$2(1-k)(2x-y) \leq 1 - k(1+y) \quad \text{and} \quad 1 - kx \leq 2(1-k)(2y-x).$$

Rearranging, we see that they are equivalent to

$$4(1-k)x - (2-3k)y \leq 1-k \quad \text{and} \quad (2-3k)x - 4(1-k)y \leq -1. \quad (36)$$

In this case, the expected period 2 price of each firm is given by

$$\begin{aligned} E\left[\sigma_A^2(p^1, d_i^1 = \emptyset, d_j^1)\right] &= \frac{1 - 2kE[e_j(h)] + 2(1-k)(y-2x)}{3} \\ &= \frac{1 - k + 2(1-k)(y-2x)}{3}, \quad \text{and} \\ E\left[\sigma_B^2(p^1, d_i^1 = \emptyset, d_j^1)\right] &= \frac{-1 + 2kE[e_j(h)] + 2(1-k)(2y-x)}{3} \\ &= \frac{-1 + k + 2(1-k)(2y-x)}{3}. \end{aligned} \quad (37)$$

[40] $\bar{q}$ is the value of $q_A = q_B$ that solves (33) and (34) with equalities. $\underline{q}$ is the value of q_B that solves (34) and (45) with equalities. q^0 is the critical value of q_A such that the period 2 equilibrium following $d_j^1 = \emptyset$ is a B-monopolization equilibrium if $q_A < q^0$, and an interior equilibrium if $q_A > q^0$.

Substituting (37) into (16) and (17), we obtain

$$\begin{aligned} p_A^1 &= (1-\delta)\left(u+1-\frac{k}{2}\right)+\frac{\delta}{3}(1-k)-(1-k)\left\{\left(1+\frac{\delta}{3}\right)x-\frac{2\delta}{3}y\right\}, \\ p_B^1 &= (1-\delta)\left(u+\frac{k}{2}\right)-\frac{\delta}{3}(1-k)+(1-k)\left\{\left(1+\frac{\delta}{3}\right)y-\frac{2\delta}{3}x\right\}. \end{aligned} \tag{38}$$

Solving (38) yields (35).

2. If $(q_A, q_B) \in R_2$, *i.e.*,

$$q^0 \le q_A \le \bar{q}, \quad q_B \ge \underline{q},$$

and

$$2\delta q_A - (3+\delta)q_B \le (1+\delta)\left\{-3+2\delta+\nu(3-\delta)\right\},$$

then

$$x = \frac{\delta - q_A + \frac{1-\delta}{1-k}(u+1-\frac{k}{2})}{1+\frac{\delta}{3}} \quad \text{and} \quad y = 1. \tag{39}$$

x is determined by the indifference condition of type x under the assumption that both $d_j^1 = A$ and $d_j^1 = \emptyset$ are followed by an interior equilibrium in period 2.

3. If $(q_A, q_B) \in R_3$, *i.e.*,

$$0 \le q_A < q^0, \quad q_B \ge \underline{q},$$

then

$$x = \frac{(3-2k)\delta - 3(1-\delta)(1-k) + \sqrt{\varphi(q_A)}}{2(4-3k)\delta} \quad \text{and} \quad y = 1, \tag{40}$$

where

$$\begin{aligned} \varphi(q_A) = &\left\{3(1-\delta)(1-k)-(3-2k)\delta\right\}^2 \\ &- 12(4-3k)\delta\left\{(1-k)q_A-(1-\delta)(u+1-\frac{k}{2})\right\}. \end{aligned} \tag{41}$$

x is determined by the indifference condition of type x under the assumption that $d_j^1 = A$ is followed by an interior equilibrium whereas $d_j^1 = \emptyset$ is followed by a B-monopolization equilibrium in period 2.

4. If $(q_A, q_B) \in R_4$, *i.e.*,

$$q^0 \le q_B \le \bar{q}, \quad q_A \ge \underline{q},$$

and

$$(3+\delta)q_A - 2\delta q_B \ge (1+\delta)\left\{\delta+(3-\delta)\mu\right\},$$

then

$$x = 0 \quad \text{and} \quad y = 1 - \frac{\delta - q_B + \frac{1-\delta}{1-k}(u + 1 - \frac{k}{2})}{1 + \frac{\delta}{3}}. \tag{42}$$

y is determined by the indifference condition of type y under the assumption that both $d_j^1 = B$ and $d_j^1 = \emptyset$ are followed by an interior equilibrium in period 2.

5. If $(q_A, q_B) \in R_5$, *i.e.*,

$$0 \le q_B \le q^0, \quad q_A \ge \underline{q},$$

then

$$x = 0, \quad \text{and} \quad y = 1 - \frac{(3-2k)\delta - 3(1-\delta)(1-k) + \sqrt{\varphi(q_B)}}{2(4-3k)\delta},$$

where φ is defined in (41). y is determined by the indifference condition of type y under the assumption that $d_j^1 = B$ is followed by an interior equilibrium while $d_j^1 = \emptyset$ is followed by an A-monopolization equilibrium in period 2.

6. If $(q_A, q_B) \in R_6$, *i.e.*,

$$\min\{q_A, q_B\} > \bar{q},$$

then

$$x = 0 \quad \text{and} \quad y = 1. \tag{43}$$

7. If $(q_A, q_B) \in R_7$, *i.e.*,

$$\max\{q_A, q_B\} < \underline{q},$$

then

$$x = y = \frac{1 - q_A + q_B}{2}. \tag{44}$$

– Beliefs:

The conditional distribution $P(\cdot \mid h)$ about buyer i's type s_i given history $h = (p^1, d^1)$ is derived through Bayes' rule if buyer i chooses d_i^1 with positive probability when faced with p^1: $P\left(\tau_i^1(s_i, p^1) = d_i^1\right) > 0$. Otherwise, $P(\cdot \mid h)$ equals the prior and is given by the uniform distribution over $[0, 1]$.

– Period 2 strategies:

Let $e_j(h) = E[s_j \mid h]$ denote the expected value of s_j according to the conditional distribution $P(\cdot \mid h)$ specified above. Then the firms' strategy profile σ^2 in period 2 is given as in Lemma 2, and each buyer's strategy is given by (5).

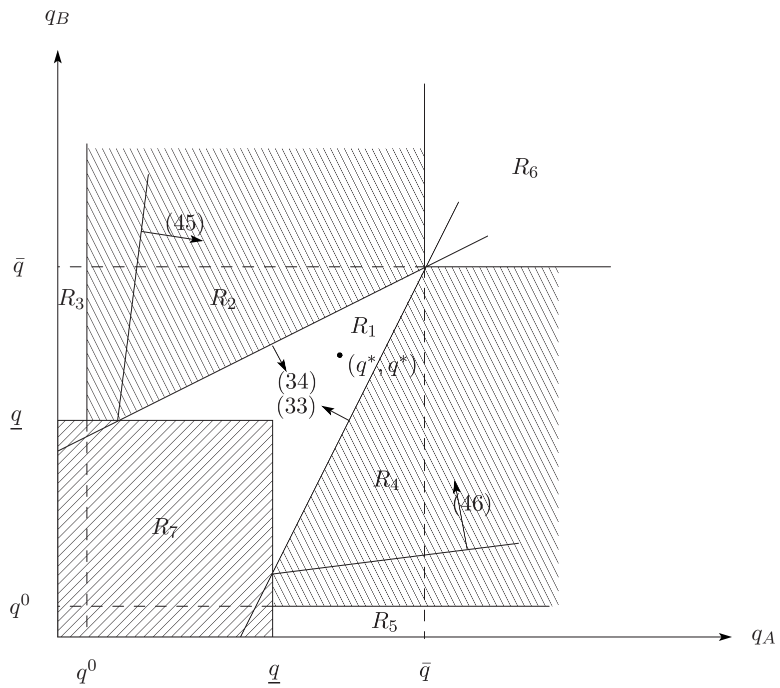

Figure 5: Classification of the period 1 price pair (q_A, q_B).

Figure 5 illustrates the classification of the period 1 price pair $(q_A, q_B) = \left(\frac{p_A^1}{1-k}, \frac{p_B^1}{1-k}\right)$ in Theorem 7. For δ close to one, the equilibrium price pair (q^*, q^*) in period 1 belongs to the interior of R_1.

It is clear from the discussion in the preceding section that the period 2 strategies of the firms and buyers are optimal. In what follows, we first show that the period 1 strategies of the buyers are optimal, and then show that the firms' period 1 price quote (31) is also optimal given the buyers' strategies. In what follows, given any price pair p^1 and decision pair d^1 in period 1, let $p_f^{2*}(d^1) = \sigma_f^2(p^1, d^1)$ denote the price quoted by firm f after history $h = (p^1, d^1)$.

Step 1. We examine the optimality of the buyers' period 1 strategies for each period 1 price profile as classified in Figure 5. We below present a proof when the price pair (q_A, q_B) belongs to region R_1 of Figure 5. The proof for the price pair (q_A, q_B) in regions R_2-R_7 can be found in the online Appendix.

1. $(q_A, q_B) \in R_1$:

   The critical types x and y are given by (35). Substituting these into the conditions (36) ensuring the interior equilibrium in period 2 after every d^1 (*i.e.*, $d^1 = (\emptyset, A)$,

$(\emptyset, \emptyset)$, and $(\emptyset, B)$), we obtain

$$\frac{12(1-k)+2\delta k}{(1+\delta)(3-\delta)} q_A + \frac{6(1-\delta)-k(9-5\delta)}{(1+\delta)(3-\delta)} q_B \geq \frac{\delta(2-k)}{3-\delta} + (1-k)(4\mu-1) - (2-3k)\nu, \tag{45}$$

and

$$\frac{6(1-\delta)-k(9-5\delta)}{(1+\delta)(3-\delta)} q_A + \frac{12(1-k)+2\delta k}{(1+\delta)(3-\delta)} q_B \geq \frac{\delta(2-k)}{3-\delta} + (1-k)(4\mu-1) - (2-3k)\nu, \tag{46}$$

As is clear from Figure 5, (q_A, q_B) under consideration satisfies these conditions when δ is close to one. The period 2 equilibrium prices are then given by (8) for each d^1, and the expected period 2 price is given by (37).

We will now examine buyer i's incentive depending on his type s_i. Note first that the following inequalities hold under (36):

$$0 < x \leq \frac{1-k(1+y)}{6(1-k)} + \frac{x+y}{3} < \frac{1-k(x+y)}{6(1-k)} + \frac{x+y}{3} < \frac{1-kx}{6(1-k)} + \frac{x+y}{3} \leq y < 1. \tag{47}$$

In the above, $s_i < x$ implies that s_i chooses A in period 1, and $s_i > y$ implies that s_i chooses B in period 1. On the other hand, Lemma 2 implies that the three quantities in the middle are the critical types s_i of buyer i who are indifferent between A and B in period 2 after buyer j's choice of $d_j^1 = B$, $\emptyset$ and A in period 1, respectively. It follows that there are the following six cases to consider depending on buyer i's decision over two periods.

- Type $s_i \leq x$ chooses A in period 1.

  Since $x > 0$, the period 1 price is chosen so that type $s_i = x$ is just indifferent between choosing A in period 1 and choosing A in period 2 after any d_j^1. It follows that any type $s_i < x$ strictly prefers the former. Any type $s_i \leq x$ also prefers the choice of A to any other choice in period 2 after any d_j^1. It follows that $s_i \leq x$ optimally chooses A in period 1.

- Type $s_i \geq y$ chooses B in period 1.

  The same logic as above shows that $s_i \geq y$ optimally chooses B in period 1.

- Type $s_i \in \left(x, \frac{1-k(1+y)}{6(1-k)} + \frac{x+y}{3}\right)$ waits in period 1 and then unconditionally chooses A in period 2.

- Type $s_i \in \left(\frac{1-k(1+y)}{6(1-k)} + \frac{x+y}{3}, \frac{1-k(x+y)}{6(1-k)} + \frac{x+y}{3}\right)$ waits in period 1 and then chooses A if $d_j^1 = A$ or $\emptyset$, and B if $d_j^1 = B$ ($\emptyset AAB$).
- Type $s_i \in \left(\frac{1-k(x+y)}{6(1-k)} + \frac{x+y}{3}, \frac{1-kx}{6(1-k)} + \frac{x+y}{3}\right)$ waits in period 1 and then chooses A if $d_j^1 = A$ and B if $\emptyset$ or $d_j^1 = B$.
- Type $s_i \in \left(\frac{1-kx}{6(1-k)} + \frac{x+y}{3}, y\right)$ waits in period 1 and then chooses B after any d_j^1.

  Any type s_i with $x < s_i < y$ strictly prefers the choice of A in period 2 after any d_j^1 to the choice of A in period 1, and the choice of B in period 2 after any d_j^1 to the choice of B in period 1. It follows that the contingent choice in period 2 as described above is optimal by the definitions of the critical types.

Step 2. We now examine the optimality of the price q^* in (31). Specifically, we show that when δ is sufficiently close to one, $q_A = q^*$ is the unique maximizer of

$$f(q_A) \equiv \hat{\Pi}_A(q_A, q^*).$$

Let

$$\begin{aligned}
z_1 &= \frac{1-\delta}{1-k}\left(u + 1 - \frac{k}{2}\right) + \frac{k\delta - 3(1-k)(1-\delta)}{4-3k}, \\
z_2 &= \frac{(1+\delta)\left\{-3 + 2\delta + \nu(3-\delta)\right\} + (3+\delta)q^*}{2\delta}, \\
z_3 &= \frac{(1+\delta)\left\{\delta + (3-\delta)\mu\right\} + 2\delta q^*}{(3+\delta)}.
\end{aligned}$$

Depending on the value of q_A, the price pair (q_A, q^*) belongs to different sets as follows:

$$(q_A, q^*) \in \begin{cases} R_3 & \text{if } q_A \in [0, z_1), \\ R_2 & \text{if } q_A \in [z_1, z_2), \\ R_1 & \text{if } q_A \in [z_2, z_3), \\ R_4 & \text{if } q_A \in [z_3, \infty). \end{cases} \tag{48}$$

As mentioned earlier, the price pair (q^*, q^*) belongs to the interior of region R_1 for δ close to one, and hence we have

$$z_2 < q^* < z_3.$$

The functional form of $\hat{\Pi}_A$ also depends on the classification of q_A in (48). We denote

$$f(q_A) = \begin{cases} f_1(q_A) & \text{if } q_A \in [0, z_1), \\ f_2(q_A) & \text{if } q_A \in [z_1, z_2), \\ f_3(q_A) & \text{if } q_A \in [z_2, z_3), \\ f_4(q_A) & \text{if } q_A \in [z_3, \infty). \end{cases}$$

Explicitly specification of each function is given below.

1. $q_A \in [0, z_1)$: x and y are as specified in (40). Since it is an expression of the indifference condition of type x when $d_j^1 = A$ is followed by an interior equilibrium and $d_j^1 = \emptyset$ is followed by a B-monopolization equilibrium, it is equivalent to

$$\begin{aligned}(1-k)q_A = (1-\delta)\left\{u + (1-k)(1-x) + \frac{k}{2}\right\} \\ + \frac{\delta x}{3}\left\{1 - kx + 2(1-k)(1-2x)\right\}.\end{aligned} \tag{49}$$

Firm A's payoff against $q_B = q^*$ is given by

$$f_1(q_A) = \hat{\Pi}_{A,1}(q_A, q^*),$$

where

$$\begin{aligned}\hat{\Pi}_{A,1}(q_A, q_B) &= (1-k)q_A x + (1-x)x\pi_A^{2*}(\emptyset, A) \\ &= (1-k)q_A x + \frac{\delta x}{18(1-k)}\left\{1 - kx + 2(1-k)(1-2x)\right\}^2.\end{aligned}$$

Using (49), we can rewrite f_1 as a function of x as:

$$\begin{aligned}\hat{f}_1(x) = (1-\delta)x\left\{u + 1 - (1-k)x - \frac{k}{2}\right\} \\ + \frac{\delta x^2}{3}\left\{1 - kx + 2(1-k)(1-2x)\right\} \\ + \frac{\delta x}{18(1-k)}\left\{1 - kx + 2(1-k)(1-2x)\right\}^2.\end{aligned} \tag{50}$$

When $q_A \in [0, z_1]$, the corresponding range of x under (40) is given by

$$x \in \left[\frac{3(1-k)}{4-3k}, \frac{(6-5k)\delta - 3(1-k) + \sqrt{\varphi(0)}}{2(4-3k)\delta}\right], \tag{51}$$

where φ is as defined in (41). The first derivative of $\hat{f}_1$ is given by

$$\hat{f}_1'(x) = (1-\delta)\left\{u + 1 - \frac{k}{2} - 2(1-k)x\right\} + \frac{\delta x}{3}\left\{2(3-2k) + 3(3k-4)x\right\}$$
$$+ \frac{\delta}{18(1-k)}\left\{3 - 2k + (3k-4)x\right\}\left\{3 - 2k + 3(3k-4)x\right\}$$

The second derivative of $\hat{f}_1$ is given by

$$\hat{f}_1''(x) = -2(1-\delta)(1-k) + \frac{\delta}{9(1-k)}\left\{-2(3-2k) + 3(3k-4)(2-3k)x\right\},$$

which is < 0 for any $x \geq 0$. We can also verify that

$$\hat{f}_1'\left(\frac{3(1-k)}{4-3k}\right) = (1-\delta)\left\{u + 1 - \frac{k}{2} - \frac{6(1-k)^2}{4-3k}\right\}$$
$$+ \frac{\delta(1-k)(5k-3)}{4-3k} + \frac{\delta k(7k-6)}{18(1-k)},$$

which is < 0 for δ close to 1. It then follows that for δ close to 1, $\hat{f}_1'(x) < 0$ for any $x \geq \frac{3(1-k)}{4-3k}$, and hence that for any such δ, $\hat{f}_1$ is maximized at the lower bound of (51). Since q_A and x are inversely related through (40), f_1 over $[0, z_1]$ is maximized at $q_A = z_1$.

2. $q_A \in [z_1, z_2)$: x and y are as specified in (39). Since it is an expression of the indifference condition of type x when $d_j^1 = A$ and $d_j^1 = \emptyset$ are both followed by an interior equilibrium in period 2, it is equivalent to

$$(1-k)q_A = (1-\delta)\left\{u + (1-k)(1-x) + \frac{k}{2}\right\} + (1-k)\frac{\delta}{3}(3-4x). \quad (52)$$

Firm A's payoff is given by

$$f_2(q_A) = \hat{\Pi}_{A,2}(q_A, q^*),$$

where

$$\begin{aligned}
&\hat{\Pi}_{A,2}(q_A, q_B)\\
&= (1-k)q_A x + \delta(1-x)\left\{x\pi_A^{2*}(\emptyset, A) + (1-x)\pi_A^{2*}(\emptyset,\emptyset)\right\}\\
&= (1-k)q_A x\\
&+ \frac{\delta}{18(1-k)}\Big[x\left\{1 - kx + 2(1-k)(1-2x)\right\}^2\\
&+ (1-x)\left\{1 - k(x+1) + 2(1-k)(1-2x)\right\}^2\Big].
\end{aligned}$$

Using (52), we can rewrite f_2 as a function of x as:

$$\begin{aligned}\hat{f}_2(x) = x &\left[(1-\delta)\left\{u + (1-k)(1-x) + \frac{k}{2}\right\} + (1-k)\frac{\delta}{3}(3-4x)\right] \\ &+ \frac{\delta}{18(1-k)}\left[x\left\{1 - kx + 2(1-k)(1-2x)\right\}^2\right. \\ &\left. + (1-x)\left\{1 - k(x+1) + 2(1-k)(1-2x)\right\}^2\right].\end{aligned}$$

When $q_A \in [z_1, z_2]$, the corresponding range of x under (39) is given by

$$x \in \left[\frac{3}{2\delta}\left\{\frac{1-\delta}{1-k}\left(u + \frac{k}{2}\right) + 1 - q^*\right\}, \frac{3(1-k)}{4-3k}\right]. \tag{53}$$

The first derivative of $\hat{f}_2$ is given by

$$\begin{aligned}\hat{f}_2'(x) = (1-\delta)&\left\{u + 1 - \frac{k}{2} - 2(1-k)x\right\} \\ &- \frac{\delta}{9(1-k)}\left\{(3k^2 - 7k + 3) + 2(5k^2 - 8k + 4)x\right\},\end{aligned}$$

which is < 0 for any $x \geq 0$ for δ close to 1. Hence, for any such δ, $\hat{f}_2$ over $[z_1, z_2]$ is maximized at the lower bound of (53). Since x and q_A are inversely related through (39), f_2 over $[z_1, z_2]$ is maximized at $q_A = z_2$.

3. $q_A \in [z_2, z_3]$: Since $(q_A, q^*) \in R_1$, substituting x and y from (35) into firm A's payoff function, we obtain

$$f_3(q_A) = \hat{\Pi}_{A,3}(q_A, q^*),$$

where $\hat{\Pi}_{A,3}(q_A, q_B) = \hat{\Pi}_A(q_A, q_B)$ is given by

$$\begin{aligned}\hat{\Pi}_A(q_A, q_B) = (1-k)q_A &\left\{\mu + \frac{\delta}{3-\delta} - \frac{(3+\delta)q_A - 2\delta q_B}{(1+\delta)(3-\delta)}\right\} \\ &+ \frac{\delta(1-k)}{18}\left[2\lambda + \frac{3(1-\delta)}{3-\delta} + \frac{6\left\{2q_A + (1-\delta)q_B\right\}}{(1+\delta)(3-\delta)}\right]^2 \\ &+ \frac{\delta k^2}{18(1-k)}\left\{\nu + \frac{\delta}{3-\delta} + \frac{(3+\delta)q_B - 2\delta q_A}{(1+\delta)(3-\delta)}\right\} \\ &\times\left\{1 - \mu - \frac{\delta}{3-\delta} + \frac{(3+\delta)q_A - 2\delta q_B}{(1+\delta)(3-\delta)}\right\} \\ &\times\left\{1 + \mu - \nu - \frac{q_A + q_B}{1+\delta}\right\}.\end{aligned} \tag{54}$$

The first-order derivative of $\hat{\Pi}_{A,3}$ with respect to q_A is given by

$$
\begin{aligned}
&\frac{\partial \hat{\Pi}_{A,3}}{\partial q_A}(q_A, q_B) \\
&= (1-k)\left\{\mu + \frac{\delta}{3-\delta} - \frac{(3+\delta)q_A - 2\delta q_B}{(1+\delta)(3-\delta)}\right\} \\
&+ (1-k)q_A\left\{-\frac{3+\delta}{(1+\delta)(3-\delta)}\right\} \\
&+ \frac{2\delta(1-k)}{18}\left\{2\lambda + \frac{3(1-\delta)}{3-\delta} + \frac{6\{2q_A + (1-\delta)q_B\}}{(1+\delta)(3-\delta)}\right\}\frac{12}{(1+\delta)(3-\delta)} \\
&+ \frac{\delta k^2}{18(1-k)}\left[\left\{\frac{-2\delta}{(1+\delta)(3-\delta)}\right\}\left\{1-\mu-\frac{\delta}{3-\delta} + \frac{(3+\delta)q_A - 2\delta q_B}{(1+\delta)(3-\delta)}\right\}\left(1+\mu-\nu-\frac{q_A+q_B}{1+\delta}\right)\right. \\
&+ \frac{3+\delta}{(1+\delta)(3-\delta)}\left\{\nu + \frac{\delta}{3-\delta} + \frac{(3+\delta)q_B - 2\delta q_A}{(1+\delta)(3-\delta)}\right\}\left(1+\mu-\nu-\frac{q_A+q_B}{1+\delta}\right) \\
&\left. + \left(\frac{-1}{1+\delta}\right)\left\{\nu + \frac{\delta}{3-\delta} + \frac{(3+\delta)q_B - 2\delta q_A}{(1+\delta)(3-\delta)}\right\}\left\{1-\mu-\frac{\delta}{3-\delta} + \frac{(3+\delta)q_A - 2\delta q_B}{(1+\delta)(3-\delta)}\right\}\right].
\end{aligned}
$$

The second-order derivative is given by

$$
\begin{aligned}
&\frac{\partial^2 \hat{\Pi}_{A,3}}{\partial q_A^2}(q_A, q_B) \\
&= -2(1-k)\,\frac{(1-\delta)(1+\delta)^2 + 8}{(1+\delta)^2(3-\delta)^2} \\
&+ \frac{\delta k^2}{9(1-k)(1+\delta)^3(3-\delta)^2}\Big[-(1+\delta)\left\{\delta(7\delta+3) + 12\delta\mu + 3(1-\delta)(3+\delta)\nu\right\} \\
&\qquad + 6\delta(3+\delta)q_A - 3(3+\delta^2)q_B\Big]
\end{aligned}
$$

Since $(3+\delta)q_A - 2\delta q_B \le (1+\delta)\{\delta + (3-\delta)\mu\}$ holds in R_1 by (33), we have

$$
\begin{aligned}
&\frac{\partial^2 \hat{\Pi}_{A,3}}{\partial q_A^2}(q_A, q_B) \\
&\le \frac{\delta k^2}{9(1-k)(1+\delta)^3(3-\delta)^2} \\
&\times \Big(-9(1-\delta^2)q_B + (1+\delta)\Big[-\delta(3+\delta) + 3(1-\delta)\left\{2\delta\mu - (3+\delta)\nu\right\}\Big]\Big).
\end{aligned}
$$

Since μ, $\nu \to 0$ as $\delta \to 1$, we conclude that for δ sufficiently close to one,

$$
\frac{\partial^2 \hat{\Pi}_{A,3}}{\partial q_A^2}(q_A, q_B) < 0 \text{ for any } (q_A, q_B) \in R_1.
$$

It follows that f_3 over $[z_2, z_3]$ is maximized at $q_A = q^*$ defined in (19).

4. $q_A \in [z_3^*, \infty)$. By (42), $x = 0$ and y is independent of q_A. It follows that $f_4(q_A)$ is a constant function, and $f_4(q_A) = f_4(z_3^*)$ for any $q_A \in [z_3^*, \infty)$.

To summarize, f_1 on $[0, z_1]$ is maximized at z_1, f_2 on $[z_1, z_2]$ is maximized at z_2, f_3 on $[z_2, z_3]$ is uniquely maximized at q^*, and f_4 on $[z_3, \infty)$ is maximized at z_3. To see that q^* is the unique maximizer of f on $\boldsymbol{R}_+$, we note that f is continuous so that $f_1(z_1) = f_2(z_1)$, $f_2(z_2) = f_3(z_2)$ and $f_3(z_3) = f_4(z_3)$.[41] This completes the proof. □

References

[1] James J. Anton, Gary Biglaiser, and Nikolaos Vettas, 2014, "Dynamic Price Competition with Capacity Constraints and a Strategic Buyer," *International Economic Review,* 55(3), 943-958.

[2] Masaki Aoyagi, 2010, "Optimal Sales Schemes against Interdependent Buyers," *American Economic Journal: Microeconomics*, 2(1), 150-182.

[3] Abhijit Banerjee, 1992, "A Simple Model of Herd Behavior," *Quarterly Journal of Economics*, 107, 797-817.

[4] David Besanko, and Wayne L. Winston, 1990, "Optimal Price Skimming by a Monopolist Facing Rational Consumers," *Management Science*, 36(5), 555-567.

[5] Manaswini Bhalla, 2012, "Dynamic Pricing under Social Learning with Strategic Consumers," Working Paper.

[6] Manaswini Bhalla, 2013, "Waterfall versus Sprinkler Product Launch Strategy: Influencing the Herd," *Journal of Industrial Economics*, 61, 138-165.

[7] Qin Bian and Sandra Forsythe, 2012, "Purchase Intention for Luxury Brands: A Cross Cultural Comparison," *Journal of Business Research,* 65(10), 1443-1451.

[8] Sushil Bikhchandani, David Hirshleifer, and Ivo Welch, 1992, "A Theory of Fads, Fashion, Custom and Cultural Change as Informational Cascades," *Journal of Political Economy*, 100, 992-1026.

[41] Between f_1 and f_2, for example, the components corresponding to the period 1 payoff both equal $(1-k)q_A x$ and hence are the same as long as q_A takes the same value z_1. The component corresponding to the period 2 payoff in f_2 is the sum of the payoffs from the interior equilibria following $d_j^1 = A$ and $d_j^1 = \emptyset$. On the other hand, since the B-monopolization equilibrium yields zero for firm A, the corresponding component in f_1 is just the payoff from the interior equilibrium following $d_j^1 = A$. However, the payoff component in f_2 from the interior equilibrium following $d_j^1 = \emptyset$ approaches zero as $q_A \to z_1$ at which the period 2 equilibrium switches from interior to B-monopolization.

[9] Subir Bose, Gerhart Orosel, Marco Ottaviani, and Lise Vesterlund, 2006, "Dynamic Monopoly Pricing and Herding," *RAND Journal of Economics*, 37, 910-928.

[10] Subir Bose, Gerhart Orosel, Marco Ottaviani, and Lise Vesterlund, 2008, "Monopoly Pricing in the Binary Herding Model," *Economic Theory*, 37, 203-41.

[11] Ramon Caminal and Xavier Vives, 1996, "Why Market Shares Matter: An Information-based Theory," *Rand Journal of Economics,* 27(2), 221-239.

[12] Ramon Caminal and Xavier Vives, 1999, "Price Dynamics and Consumer Learning," *Journal of Economics and Management Strategy,* 8(1), 95-131.

[13] Christophe Chamley, 2004, *Rational Herds: Economic Models of Social Learning,* Cambridge University Press: Cambridge.

[14] Christophe Chamley and Douglas Gale, 1994, "Information Revelation and Strategic Delay in a Model of Investment," *Econometrica,* 62, 1065-1085.

[15] Yuxin Chen, and Z. John Zhang, 2009, "Dynamic Targeted Pricing with Strategic Consumers," *International Journal of Industrial Organization*, 27, 43-50.

[16] Jérémy Gallien, 2006, "Dynamic Mechanism Design for Online Commerce," *Operations Research,* 54(2) 291-310.

[17] Jochen Gönsch, Robert Klein, Michael Neugebauer, and Claudius Steinhardt, 2012, "Dynamic Pricing with Strategic Consumers," *Journal of Business Economics*, 83(5), 505-549.

[18] Faruk Gul and Russell J. Lundholm, 1995, "Endogenous Timing and the Clustering of Agents' Decisions," *Journal of Political Economy,* 103(5), 1039-1066.

[19] Hikmet Gunay, 2008, "The Role of Externalities and Information Aggregation in Market Collapse," *Economic Theory*, 35, 367-379.

[20] Hikmet Gunay, 2008, "Information Aggregation under Strategic Delay," *Economics Bulletin,* 12(23), 1-8.

[21] Hikmet Gunay, 2014, "Waiting for Signaling Quality," *Southern Economic Journal,* 81, 364-386.

[22] Philip Jehiel and Benny Moldovanu, 2001, "Efficient Design with Interdependent Valuations," *Econometrica,* 69(5), 1237-1259.

[23] Bing Jing, 2011, "Social learning and dynamic pricing of durable goods," *Marketing Science,* 30(5), 851-865.

[24] Yuri Levin, Jeff McGill, and Mikhail Nediak, 2009, "Dynamic Pricing in the Presence of Strategic Consumers and Oligopolistic Competition," *Management Science*, 55(1), 32-46.

[25] Qian Liu, and Dan Zhang, 2013, "Dynamic Pricing Competition with Strategic Customers under Vertical Product Differentiation," *Management Science*, 59(1), 84-101.

[26] Vincent Mak, Amnon Rapoport, and Eyran J. Gisches, 2012, "Competitive Dynamic Pricing with Alternating Offers: Theory and Experiment," *Games and Economic Behavior*, 75, 250-264.

[27] Daniel Sgroi, 2002, "Optimizing Information in the Herd: Guinea Pigs, Profits and Welfare," *Games and Economic Behavior,* 39, 137-166.

[28] Paurav Shukla, 2010, "Status Consumption in Cross-national Context: Socio-psychological, Brand and Situational Antecedents," *International Marketing Review,* 27(1), 108-129.

[29] Donald M. Topkis, 1998, *Supermodularity and Complementarity,* Princeton, NJ: Princeton University Press.

[30] Robert J. Weber, 1981, "Multiple-Object Auctions," Discussion Paper 496, Northwestern University.

[31] Keith Wilcox, Hyeong Min Kim, Sankar Sen, 2009, "Why Do Consumers Buy Counterfeit Luxury Brands?," *Journal of Marketing Research,* 46(2), 247-259.

[32] Man Yu, Roman Kapuscinski, and Hyun-Soo Ahn, 2015, "Advance Selling: Effects of Interdependent Consumer Valuations and Seller 's Capacity," Article in Advance, *Management Science.*

B Online Appendix

Proof of Lemma 2. Since the conditional probability $P(\cdot \mid h)$ of s_i given $h \in H_i$ is the uniform distribution over the interval (x, y), firm A's payoff from buyer i in period 2 is explicitly given by:

$$\pi^2_{A,i}(p^2 \mid \tau^2_i, h) = \begin{cases} \frac{p^2_A}{y-x}\left(\frac{1-2ke_j(h)-p^2_A+p^2_B}{2(1-k)} - x\right) & \\ \qquad \text{if } \frac{u+1-ke_j(h)-p^2_A}{1-k} \geq \frac{1-2ke_j(h)-p^2_A+p^2_B}{2(1-k)} \in (x,y), & \\ \frac{p^2_A}{y-x}\left(\frac{u+1-ke_j(h)-p^2_A}{1-k} - x\right) & \\ \qquad \text{if } \frac{1-2ke_j(h)-p^2_A+p^2_B}{2(1-k)} \geq \frac{u+1-ke_j(h)-p^2_A}{1-k} \in (x,y), & \\ p^2_A & \text{if } \min\left\{\frac{1-2ke_j(h)-p^2_A+p^2_B}{2(1-k)}, \frac{u+1-ke_j(h)-p^2_A}{1-k}\right\} \geq y, \\ 0 & \text{if } \min\left\{\frac{1-2ke_j(h)-p^2_A+p^2_B}{2(1-k)}, \frac{u+1-ke_j(h)-p^2_A}{1-k}\right\} \leq x, \end{cases}$$

and firm B's payoff from buyer i is given by:

$$\pi^2_{B,i}(p^2 \mid \tau^2, h) = \begin{cases} \frac{p^2_B}{y-x}\left(y - \frac{1-2ke_j(h)-p^2_A+p^2_B}{2(1-k)}\right) & \\ \qquad \text{if } \frac{-u-ke_j(h)+p^2_B}{1-k} \leq \frac{1-2ke_j(h)-p^2_A+p^2_B}{2(1-k)} \in (x,y), & \\ \frac{p^2_B}{y-x}\left(y - \frac{-u-ke_j(h)+p^2_B}{1-k}\right) & \\ \qquad \text{if } \frac{1-2ke_j(h)-p^2_A+p^2_B}{2(1-k)} \leq \frac{-u-ke_j(h)+p^2_B}{1-k} \in (x,y), & \\ 0 & \text{if } \max\left\{\frac{1-2ke_j(h)-p^2_A+p^2_B}{2(1-k)}, \frac{-u-ke_j(h)+p^2_B}{1-k}\right\} \geq y, \\ p^2_B & \text{if } \max\left\{\frac{1-2ke_j(h)-p^2_A+p^2_B}{2(1-k)}, \frac{-u-ke_j(h)+p^2_B}{1-k}\right\} \leq x. \end{cases}$$

We assume in the rest of the proof that $u > 2(1-k)$ to avoid tedious case separation in the description of the best response that is immaterial to the description of the equilibrium.[42]

[42] This condition ensures that the intersection between $p^2_A - p^2_B = 1 - 2ke_j(h) - 2(1-k)x$ and $p^2_B = \frac{-1+2ke_j(h)+2(1-k)y+p^2_A}{2}$ given by

$$(p^2_A, p^2_B) = (1 - 2ke_j(h) + 2(1-k)(y-2x), 2(1-k)(y-x)),$$

and the intersection between $p^2_A - p^2_B = 1 - 2ke_j(h) - 2(1-k)y$ and $p^2_A = \frac{1-2ke_j(h)-2(1-k)x+p^2_B}{2}$ given by

$$(p^2_A, p^2_B) = (2(1-k)(y-x), -1 + 2ke_j(h) + 2(1-k)(2y-x)),$$

are both below the participation constraint line $p^2_A + p^2_B = 2u + 1$ so that the diagram is as depicted in Figure 6. The condition $u > 1-k$ implied by this ensures that the maximum of $\pi^2_A(p^2 \mid \tau^2_i, h)$ over R_2 is achieved at the left-end of the region at $p^2_A = u + 1 - ke_j(h) - (1-k)y$, and also that the maximum of $\pi^2_A(p^2 \mid \tau^2_i, h)$ over the corresponding set is achieved at the lower-end of the region at $p^2_B = u + ke_j(h) + (1-k)x$ so that the best response functions are as described in Figure 6.

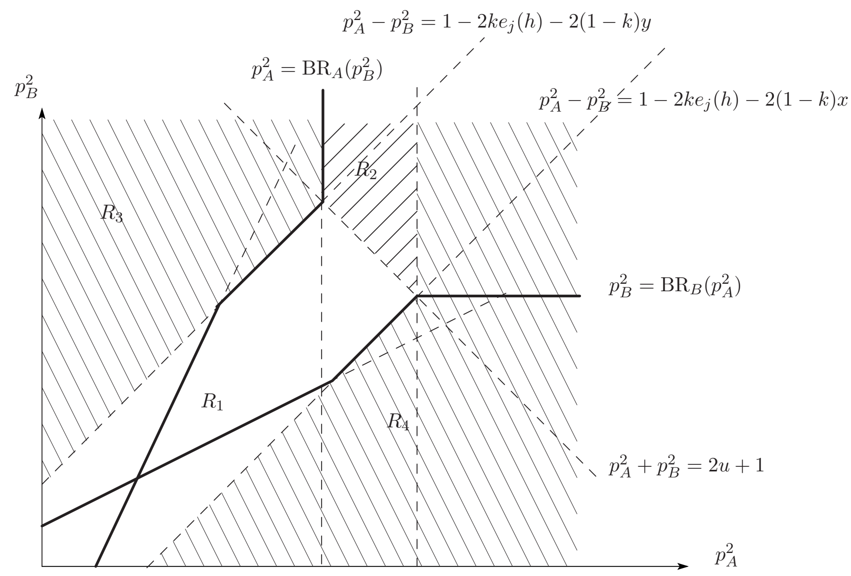

Figure 6: Best-response diagram in period 2: interior equilibrium

Let $R_1, \ldots, R_4$ be the sets of price profiles (p_A^2, p_B^2) as illustrated in Figure 6. Explicitly, they are the set of (p_A^2, p_B^2) satisfying $p_A^2,\, p_B^2 \geq 0$, and

$$\begin{aligned}
R_1: &\quad 1 - 2ke_j(h) - 2(1-k)y \leq p_A^2 - p_B^2 \leq 1 - 2ke_j(h) - 2(1-k)x, \\
&\quad p_A^2 + p_B^2 \leq 2u + 1; \\
R_2: &\quad u + 1 - ke_j(h) - (1-k)y \leq p_A^2 \leq u + 1 - ke_j(h) - (1-k)x, \\
&\quad p_A^2 + p_B^2 \geq 2u + 1; \\
R_3: &\quad p_A^2 < u + 1 - ke_j(h) - (1-k)y, \\
&\quad p_A^2 - p_B^2 < 1 - 2ke_j(h) - 2(1-k)y; \\
R_4: &\quad p_A^2 > \min\left\{p_B^2 + 1 - 2ke_j(h) - 2(1-k)x,\, u + 1 - ke_j(h) - (1-k)x\right\}.
\end{aligned}$$

We can express $\pi_{A,i}^2(p^2 \mid \tau_i^2, h)$ in terms of these sets as

$$\pi_{A,i}^2(p^2 \mid \tau_i^2, h) = \begin{cases}
\frac{p_A^2}{y-x}\left(\frac{1-2ke_j(h)-p_A^2+p_B^2}{2(1-k)} - x\right) & \text{if } (p_A^2, p_B^2) \in R_1, \\
\frac{p_A^2}{y-x}\left(\frac{u+1-ke_j(h)-p_A^2}{1-k} - x\right) & \text{if } (p_A^2, p_B^2) \in R_2, \\
p_A^2 & \text{if } (p_A^2, p_B^2) \in R_3, \\
0 & \text{if } (p_A^2, p_B^2) \in R_4.
\end{cases}$$

It follows that firm A's period 2 best response correspondence is given by

$$\mathrm{BR}_A(p_B^2) = \begin{cases} \boldsymbol{R}_+ & \text{if } 0 \leq p_B^2 < \max\{0,\, -1 + 2ke_j(h) + 2(1-k)x\}, \\ \left\{\frac{1-2ke_j(h)-2(1-k)x+p_B^2}{2}\right\} & \\ & \text{if } p_B^2 \geq \max\{0,\, -1+2ke_j(h)+2(1-k)x\}, \text{ and} \\ & p_B^2 < \max\{0,\, -1+2ke_j(h)+2(1-k)(2y-x)\}, \\ \left\{1-2ke_j(h)-2(1-k)y+p_B^2\right\} & \\ & \text{if } p_B^2 \geq \max\{0,\, -1+2ke_j(h)+2(1-k)(2y-x)\}, \text{ and} \\ & p_B^2 \leq u + ke_j(h) + (1-k)y, \\ \left\{u+1-ke_j(h)-(1-k)y\right\} & \text{if } p_B^2 > u + ke_j(h) + (1-k)y. \end{cases}$$

Likewise, firm B's period 2 best response correspondence is given by

$$\mathrm{BR}_B(p_A^2) = \begin{cases} \boldsymbol{R}_+ & \text{if } 0 \leq p_A^2 < \max\{1-2ke_j(h)-2(1-k)y,\, 0\}, \\ \left\{\frac{-1+2ke_j(h)+2(1-k)y+p_A^2}{2}\right\} & \\ & \text{if } p_A^2 \geq \max\{1-2ke_j(h)-2(1-k)y,\, 0\}, \text{ and} \\ & p_A^2 < \max\{0,\, 1-2ke_j(h)-2(1-k)(2x-y)\}, \\ \left\{-1+2ke_j(h)+2(1-k)x+p_A^2\right\} & \\ & \text{if } p_A^2 \geq \max\{0,\, 1-2ke_j(h)-2(1-k)(2x-y)\}, \text{ and} \\ & p_A^2 < u+1-ke_j(h)-(1-k)x, \\ \left\{u+ke_j(h)+(1-k)x\right\} & \text{if } p_B^2 \geq u+1-ke_j(h)-(1-k)x. \end{cases}$$

Figure 6 depicts these best response correspondences for the case $2(1-k)x < 1 - 2ke_j(h) < 2(1-k)y$. Note also that when $p_A^2 - p_B^2 \leq 1 - 2ke_j(h) - 2(1-k)y$, firm A monopolizes the market under (p_A^2, p_B^2), and that when $p_A^2 - p_B^2 \geq 1 - 2ke_j(h) - 2(1-k)x$, firm B monopolizes the market under (p_A^2, p_B^2). Note also that the participation constraint does not bind for the critical type that is indifferent between firms A and B if $p_A^2 + p_B^2 < 2u + 1$.

a) $1 - 2ke_j(h) \in [2(1-k)(2x-y),\, 2(1-k)(2y-x)]$.

The best response correspondences $p_A^2 = \mathrm{BR}_A(p_B^2)$ and $p_B^2 = \mathrm{BR}_B(p_A^2)$ have a unique intersection

$$\left(\frac{1-2ke_j(h)+2(1-k)(y-2x)}{3},\, \frac{-1+2ke_j(h)+2(1-k)(2y-x)}{3}\right),$$

which satisfies $1 - 2ke_j(h) - 2(1-k)x < p_A^2 - p_B^2 < 1 - 2ke_j(h) - 2(1-k)y$ and also $p_A^2 + p_B^2 < 2u + 1$ when $u > \frac{1}{2} - k$. Hence, the two firms segment the market and the critical type is given by $s_i = \frac{1-2ke_j(h)}{6(1-k)} + \frac{x+y}{3}$.

b) $1 - 2ke_j(h) > 2(1-k)(2y - x)$.

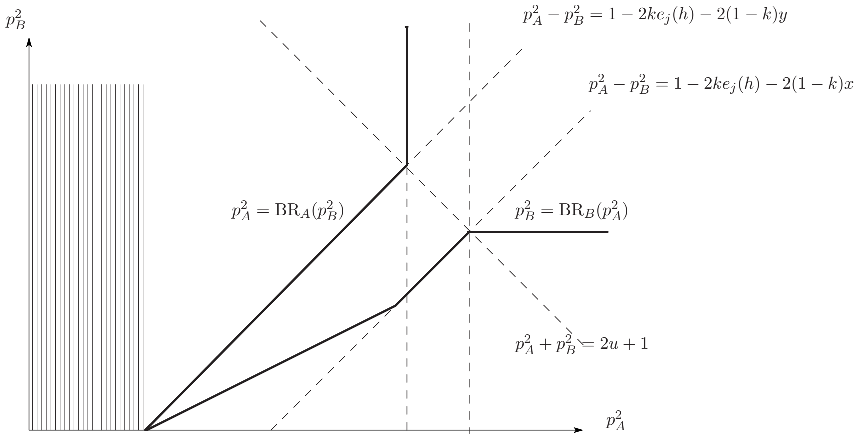

Figure 7: Best-response diagram: A-monopolization equilibrium

As seen in Figure 7, the unique fixed point of the joint best-response correspondence $(p_A^2, p_B^2) \twoheadrightarrow \big(\mathrm{BR}_A(p_B^2),\, \mathrm{BR}_B(p_A^2)\big)$ is given by

$$(1 - 2ke_j(h) - 2(1-k)y,\, 0)\,.$$

Since $p_A^2 - p_B^2 = 1 - 2ke_j(h) - 2(1-k)y$, firm A monopolizes the market.

c) $1 - 2ke_j(h) < 2(1-k)(2x - y)$.

As in the previous case, the unique fixed point of the joint best-response correspondence $(p_A^2, p_B^2) \twoheadrightarrow \big(\mathrm{BR}_A(p_B^2),\, \mathrm{BR}_B(p_A^2)\big)$ is given by

$$(0,\, -1 + 2ke_j(h) + 2(1-k)x)\,.$$

Since $p_A^2 - p_B^2 \geq 1 - 2ke_j(h) - 2(1-k)x$, firm B monopolizes the market.

This completes the proof. □

Proof of Corollary 4. Fix p^1 and let $h = (p^1, d^1) \in H_i$. Suppose to the contrary that $p_A^1 \leq E[\sigma_A^2(h)] = E[\sigma_A^2(h) \mid s_i = x]$. It will then follow from the first equation in Lemma 3 that

$$E\left[v_i - \sigma_A^2(h) \mid s_i = x\right] \leq \delta\, E\left[v_i - \sigma_A^2(h) \mid s_i = x\right].$$

Since $\delta < 1$, this implies that

$$E\left[v_i - \sigma_A^2(h) \mid s_i = x\right] \leq 0.$$

In other words, the expected payoff of type x is non-positive when he always chooses A in period 2. We will derive a contradiction by showing that type x's payoff in any period 2 equilibrium is strictly positive when $x < y$.

Suppose first that an interior equilibrium is played in period 2 following history h. In this case, type x must obtain a strictly positive payoff by choosing A since any type $s_i > x$ who chooses A also obtains a non-positive payoff. Formally,

$$\begin{aligned}
&\text{type } x\text{'s payoff from choosing } A \\
&= u + 1 - (1-k)x - ke_j - \frac{1}{3}\left\{1 - 2ke_j(h) + 2(1-k)(y-2x)\right\} \\
&= u + \frac{2}{3} - \frac{k}{3}e_j(h) - \frac{1}{3}(1-k)(2y-x) \\
&\geq u + \frac{2}{3} - \frac{k}{3} - \frac{2}{3}(1-k) \\
&= u + \frac{k}{3} > 0.
\end{aligned}$$

Suppose next that we have the A-monopolization equilibrium in period 2. In this case,

$$\begin{aligned}
&\text{type } x\text{'s payoff from choosing } A \\
&= u + 1 - (1-k)x - ke_j(h) - \left\{1 - 2ke_j(h) - 2(1-k)y\right\} \\
&= u + (1-k)(2y-x) + ke_j(h) > 0.
\end{aligned}$$

Suppose finally that we have the B-monopolization equilibrium in period 2. In this case,

$$\begin{aligned}
&\text{type } x\text{'s payoff from choosing } A \\
&= u + 1 - (1-k)x - ke_j(h) - 0 > 0.
\end{aligned}$$

This completes the proof. □

Proof of Theorem 7. (continued) As part of the proof of Theorem 7, we show that the buyers' period 1 strategies are sequentailly rational following (off-equilibrium) price pairs that belong to regions R_2-R_7 in Figure 5.

2. $(q_A, q_B) \in R_2$.

   Since $y = 1$, $d_j^1 = B$ occurs with probability zero. For x and y given in (39), $d^1 = (\emptyset, A)$ and $(\emptyset, \emptyset)$ are both followed by an interior equilibrium in period 2. Hence,

   $$0 < x \leq \frac{1 - k(x+1)}{6(1-k)} + \frac{x+1}{3} < \frac{1 - kx}{6(1-k)} + \frac{x+1}{3} \leq y = 1.$$

The two values in the middle are the critical types of buyer i who are indifferent between A and B in period 2 after $d_j^1 = \emptyset$ and $d_j^1 = A$, respectively. Since $x > 0$, any type $s_i < x$ strictly prefers the choice of A in period 1 to the unconditional choice of A in period 2, and the reverse is true for any type $s_i > x$. The optimality of the contingent choice of the goods in period 2 then would follow if any type $s_i \leq 1$ prefers the unconditional choice of B in period 2 to the choice of B in period 1. Suppose that $s_i = 1$. Then the choice of B in period 1 yields

$$u + (1-k) \cdot 1 + \frac{k}{2} - (1-k)q_B,$$

whereas the unconditional choice of B in period 2 yields

$$\delta \left\{ u + (1-k) \cdot 1 + \frac{k}{2} - (1-k)\frac{3-2x}{3} \right\},$$

where $(1-k)\frac{3-2x}{3}$ is the expected price of B in period 2. Hence, type $s_i = 1$ prefers the choice of B in period 2 if

$$(1-k)\left\{ q_B - \delta\frac{3-2x}{3} \right\} \geq (1-\delta)\left(u + 1 - \frac{k}{2} \right).$$

Substituting for x from (39) and for ν from (20), we see that this is equivalent to

$$(3+\delta)q_B - 2\delta q_A \geq \delta(3-\delta) + \frac{(1-\delta)(3-\delta)}{1-k}\left(u + 1 - \frac{k}{2} \right),$$

which holds by definition for any $(q_A, q_B) \in R_2$. Hence, type $s_i = 1$ prefers the choice of B in period 2, and so does any type $s_i < 1$.

3. $(q_A, q_B) \in R_3$.

   Since $y = 1$, $d_j^1 = B$ occurs with probability zero. For x and y given in (40), $(d_i^1, d_j^1) = (\emptyset, \emptyset)$ is followed by a B-monopolization equilibrium and $(d_i^1, d_j^1) = (\emptyset, A)$ is followed by an interior equilibrium in period 2.

   We first show that for δ close to one, any type s_i prefers the unconditional choice of B in period 2 to the choice of B in period 1. To see this, note that the expected price of B in period 2 equals

$$\begin{aligned}
&E\left[p_B^{2*}(d^1)\right] \\
&= x p_B^{2*}(\emptyset, A) + (1-x) p_B^{2*}(\emptyset, \emptyset) \\
&= x\frac{-1+kx+2(1-k)(2-x)}{3} + (1-x)\left\{-1+k(1+x)+2(1-k)x\right\} \\
&= \frac{-3(1-k)+2x(6-5k)-2x^2(4-3k)}{3} \\
&= -\frac{2}{3}(4-3k)\left\{x - \frac{6-5k}{2(4-3k)}\right\}^2 + \frac{(6-5k)^2}{6(4-3k)} - (1-k) \\
&\leq \frac{(6-5k)^2}{6(4-3k)} - (1-k).
\end{aligned}$$

On the other hand, since $p_B^1 = (1-k)q_B \geq (1-k)\underline{q}$ and $\underline{q} \to \frac{3-2k}{6(1-k)}$ as $\delta \to 1$, the expected price of B in period 2 is lower than p_B^1 for δ close to one if

$$\frac{(6-5k)^2}{6(4-3k)} - (1-k) < \frac{3-2k}{6},$$

which holds since $k < \frac{1}{2}$. It follows that for δ close to one, any type s_i prefers the choice of B in period 2 to the choice of B in period 1. Since $x > 0$, type $s_i < x$ prefers the choice of A in period 1 to the unconditional choice of A in period 2, and the reverse is true for any type $s_i > x$. Furthermore, since $x < \frac{3-2k}{4-3k}$,

$$\frac{1-k(x+1)}{6(1-k)} + \frac{x+1}{3} < x < \frac{1-kx}{6(1-k)} + \frac{x+1}{3} \leq y = 1.$$

The optimality of the contingent choice of the good in period 2 then follows from the definitions of the critical types.

4. $(q_A, q_B) \in R_4$: This case is similar to when $(q_A, q_B) \in R_2$.

5. $(q_A, q_B) \in R_5$: This case is similar to when $(q_A, q_B) \in R_3$.

6. $(q_A, q_B) \in R_6$.

   Every type waits under the given specifications of x and y ($x = 0$ and $y = 1$). The equilibrium price pair in period 2 following $d_i = (\emptyset, \emptyset)$ then equals $(1-k, 1-k)$. The conclusion would follow if we show that any type prefers waiting to moving in period 1 when q_A and q_B are both at the lowest level consistent with $(q_A, q_B) \in R_6$:

$$q_A = q_B = \frac{1+\delta}{3-\delta}\left\{3 - 2\delta - \nu(3-\delta)\right\}.$$

Take any type $s_i < \frac{1}{2}$ who prefers A to B in period 1. If he chooses A in period 1, then $E[v_i \mid s_i] - (1-k)q_A$. If he waits, it yields $\delta E[v_i \mid s_i] - (1-k)$. Hence, waiting is optimal if

$$q_A = \frac{1+\delta}{3-\delta}\{3 - 2\delta - \nu(3-\delta)\} > \frac{1-\delta}{1-k}(u + \frac{1}{3}) + \delta.$$

Substituting the definition of ν from (20) and simplifying, we see that this is equivalent to $\delta < 3$, which holds.

7. $(q_A, q_B) \in R_7$.

No type waits under the given specification of x and y ($x = y = \frac{1-q_A+q_B}{2}$.) By construction, the conditional belief $P(\cdot \mid h)$ of s_i given $h \in H_i$ is the uniform distribution over $[0,1]$. Hence, any buyer who waits will face the price pair $(1-k, 1-k)$ in period 2. Consider any type $s_i < x$. He prefers A to B in period 1. For δ close to one, he also prefers the choice of A in period 1 to the unconditional choice of A in period 2 since the price of A in period 2 is higher: $q_A \leq \underline{q}$ and as $\delta \to 1$,

$$(1-k)\underline{q} \to \frac{3-2k}{6} < 1-k.$$

If he waits and then chooses A after $d_j^1 = A$ and B after $d_j^1 = B$, then his payoff is given by

$$\begin{aligned}
&\delta\left\{xE\left[v_i \mid s_i, p^1, d_j^1 = A\right] + (1-x)E\left[w_i \mid s_i, p^1, d_j^1 = B\right] - (1-k)\right\} \\
&= \delta\Bigg[x\left\{u + 1 - (1-k)s_i - k\frac{x}{2}\right\} \\
&\qquad + (1-x)\left\{u + (1-k)s_i + k\frac{1+x}{2}\right\} - (1-k)\Bigg] \\
&= \delta\left\{u + x - (1-k) + (1-k)s_i(1-2x) + \frac{k}{2}(1-2x^2)\right\}.
\end{aligned}$$

On the other hand, choosing A in period 1 yields

$$E[v_i \mid s_i] - (1-k)q_A \geq u + 1 - (1-k)s_i - \frac{k}{2} - (1-k)\underline{q}.$$

Choosing A in period 1 is hence optimal for type $s_i \leq x$ if

$$\begin{aligned}
&\delta\left\{u + x - (1-k) + (1-k)s_i(1-2x) + \frac{k}{2}(1-2x^2)\right\} \\
&\leq u + 1 - (1-k)s_i - \frac{k}{2} - (1-k)\underline{q}.
\end{aligned}$$

Since $\underline{q} \to \frac{3-2k}{6(1-k)}$ as $\delta \to 1$, this inequality holds for δ close to one if

$$\begin{aligned} & u + x - (1-k) + (1-k)s_i(1-2x) + \frac{k}{2}(1-2x^2) \\ & \leq u + 1 - (1-k)s_i - \frac{k}{2} - \frac{3-2k}{6}. \end{aligned}$$

We also see that if this inequality holds for $s_i = x$, then it holds for any $s_i \leq x$. Substituting $s_i = x$ and simplifying, we rewrite this inequality as

$$(2-k)\left\{x - \frac{3-2k}{2(2-k)}\right\}^2 - \frac{(3-2k)^2}{4(2-k)} + \frac{3}{2} - \frac{5k}{3} > 0.$$

This holds true for any x if

$$\frac{3}{2} - \frac{5k}{3} > \frac{(3-2k)^2}{4(2-k)} \quad \Leftrightarrow \quad (1-2k)(9-4k) > 0.$$

Hence, type x prefers the choice of A in period 1 to waiting. The symmetric argument proves that choosing B in period 1 is optimal when $s_i > y$. □

Proof of Proposition 9. In the proof, we consider a slightly more general setup where the prices are fixed but are not necessarily equal to the marginal cost or the same across the two periods. Specifically, consider the symmetric price profile such that $p_A^1 = p_B^1$ in period 1 and $p_A^2 = p_B^2$ in period 2. If we write $x = x(p^1)$ and $y = y(p^1)$ in (3), then by symmetry, $y = 1 - x$. Let Δ be the difference between the period 1 price and the discounted period 2 price:

$$\Delta = p_A^1 - \delta p_A^2 = p_B^1 - \delta p_B^2.$$

Let $\overline{\Delta}$ as defined in (22) and

$$\begin{aligned} \underline{\Delta} &= \overline{\Delta} - (1-\delta)\frac{(1-k)(1-2k)}{2-3k}, \quad \text{and} \\ \widetilde{\Delta} &= \overline{\Delta} + \frac{\delta(2-3k)}{4} - \frac{1-k}{2}. \end{aligned}$$

Define also

$$\hat{x} = \frac{\overline{\Delta} - \Delta}{(1-\delta)(1-k)}, \tag{55}$$

and

$$\bar{x} = \frac{\delta(2-3k) - (1-k) + \sqrt{\{\delta(2-3k) - (1-k)\}^2 + 4\delta(2-3k)(\overline{\Delta} - \Delta)}}{2\delta(2-3k)}. \tag{56}$$

The buyer behavior along a fixed price path can then be described as follows.

Proposition 10. *(Buyer behavior along a fixed price path) Suppose that the price profile* (p^1, p^2) *is symmetric and fixed. Let* $\Delta = p_A^1 - \delta p_A^2$. *Then for* $\hat{x}$ *and* $\bar{x}$ *defined in (55) and (56), we have*

$$x = 1 - y = \begin{cases} 0 & \text{if } \Delta > \overline{\Delta}, \\ \hat{x} & \text{if } \Delta \in (\underline{\Delta}, \overline{\Delta}], \\ \bar{x} & \text{if } \Delta \in (\widetilde{\Delta}, \underline{\Delta}], \\ \frac{1}{2} & \text{if } \Delta \leq \widetilde{\Delta}. \end{cases}$$

In every case, any $s_i \in (x, \frac{1}{2})$ *chooses* A *in period 2 if* $d_j^1 = A$ *or* $\emptyset$. *When* $d_j^1 = B$, *then the period 2 choice of any* $s_i \in (x, \frac{1}{2})$ *is: (i)* A *in the first case* ($\Delta > \overline{\Delta}$), *(ii)* A *if* $s_i \in \left(\hat{x}, \frac{1-k(2-\hat{x})}{2(1-k)}\right)$ *and* B *if* $s_i \in \left(\frac{1-k(2-\hat{x})}{2(1-k)}, \frac{1}{2}\right)$ *in the second case* ($\Delta \in (\underline{\Delta}, \overline{\Delta}]$), *(iii)* B *in the third case* ($\Delta \in (\widetilde{\Delta}, \underline{\Delta}]$).

Since $\widetilde{\Delta} < 0$ for δ close to one, when both firms engage in marginal cost pricing $p_A^1 = p_B^1 = 0$ and $p_A^2 = p_B^2 = 0$, we have $\Delta = 0 \in (\widetilde{\Delta}, \underline{\Delta})$, and $\bar{x} = x^0$ defined in (21). Proposition 10 hence implies Proposition 9.

Proof of Proposition 10. It can be readily verified that $\max\{0, \widetilde{\Delta}\} < \underline{\Delta} < \overline{\Delta}$. By symmetry, any type $s_i < \frac{1}{2}$ prefers the choice of A in period 1 to the choice of B in period 1. Furthermore, since $x = 1 - y$, any such type s_i prefers A to B in period 2 also if the other buyer chooses A or waits in period 1. Hence, any buyer type $s_i < \frac{1}{2}$ may optimally adopt one of the following three courses of action: (i) 1A: "choose A in period 1", (ii) 2A: "wait and choose A after any d_j^1", or (iii) 2B: "wait and choose B if and only if $d_j^1 = B$." First, note that 1A and 2A yield

$$u + 1 - \frac{k}{2} - (1-k)s_i - p_A^1 \quad \text{and} \quad \delta\left[u + 1 - \frac{k}{2} - (1-k)s_i - p_A^2\right],$$

respectively. Then $x = \hat{x}$ in (55) is the type that is indifferent between 1A and 2A. In particular, if $\Delta > \overline{\Delta}$, then every type $s_i < \frac{1}{2}$ prefers 2A to 1A and hence $x = 0$. On the other hand, 2B yields

$$\begin{aligned} &\delta\Big[x\left\{u + 1 - (1-k)s_i - k\frac{x}{2} - p_A^2\right\} \\ &+ (1-2x)\left\{u + 1 - (1-k)s_i - k\frac{1}{2} - p_A^2\right\} \\ &+ x\left\{u + (1-k)s_i + k\left(1 - \frac{x}{2}\right) - p_B^2\right\}\Big], \end{aligned} \tag{57}$$

where the three terms correspond to i's choice of A, A, and B in period 2 when j's decision in period 1 is A, $\emptyset$, and B, respectively. Let $\bar{x}$ be the type that is indifferent between 1A and 2B. We see that $\bar{x}$ satisfies

$$\delta(2-3k)\bar{x}^2 - \{\delta(2-3k) - (1-k)\}\,\bar{x} - \overline{\Delta} + \Delta = 0. \tag{58}$$

A solution to (58) exists in $\left[0, \frac{1}{2}\right]$ if and only if

$$\widetilde{\Delta} \leq \Delta \leq \overline{\Delta}. \tag{59}$$

In this case, $\bar{x}$ is given by in (56).

Type s_i prefers strategy 2B to 2A if

$$s_i > \frac{1-k(2-x)}{2(1-k)}.$$

Hence, if

$$x > \frac{1-k(2-x)}{2(1-k)} \quad \Leftrightarrow \quad x > \frac{1-2k}{2-3k},$$

then every type $s_i > x$ prefers 2B to 2A. Conversely, if $x < \frac{1-2k}{2-3k}$, then type $s_i \in \left(x, \frac{1-k(2-x)}{2(1-k)}\right)$ prefers 2A to 2B and type $s_i \in \left(\frac{1-k(2-x)}{2(1-k)}, \frac{1}{2}\right)$ prefers 2B to 2A.

Suppose first that $\Delta > \underline{\Delta}$. We can verify that

$$\begin{aligned}
\Delta > \underline{\Delta} \quad &\Leftrightarrow \quad \hat{x} < \frac{1-2k}{2-3k} \\
&\Leftrightarrow \quad \delta(2-3k)\hat{x}^2 - \{\delta(2-3k) - (1-k)\}\,\hat{x} - \overline{\Delta} + \Delta < 0 \\
&\Leftrightarrow \quad \hat{x} < \bar{x}.
\end{aligned}$$

Hence, type $s_i < \hat{x}$ prefers 1A to 2A to 2B, type $s_i \in \left(\hat{x}, \frac{1-k(2-\hat{x})}{2(1-k)}\right)$ prefers 2A to 1A and 2A to 2B, and type $s_i \in \left(\frac{1-k(2-\hat{x})}{2(1-k)}, \frac{1}{2}\right)$ prefers 2B to 2A to 1A. It follows that when $\Delta > \underline{\Delta}$, the optimal course of action is given by 1A if $s_i < \hat{x}$, 2A if $s_i \in \left(\hat{x}, \frac{1-k(2-\hat{x})}{2(1-k)}\right)$, and 2B if $\left(\frac{1-k(2-\hat{x})}{2(1-k)}, \frac{1}{2}\right)$.

Suppose next that $\Delta < \underline{\Delta}$. We can verify that

$$\begin{aligned}
\Delta < \underline{\Delta} \quad &\Leftrightarrow \quad \delta(2-3k)\left(\frac{1-2k}{2-3k}\right)^2 - \{\delta(2-3k) - (1-k)\}\left(\frac{1-2k}{2-3k}\right) - \overline{\Delta} + \Delta < 0 \\
&\Leftrightarrow \quad \bar{x} > \frac{1-2k}{2-3k},
\end{aligned}$$

and also that

$$\Delta < \underline{\Delta} \quad \Leftrightarrow \quad \delta(2-3k)\hat{x}^2 - \{\delta(2-3k) - (1-k)\}\,\hat{x} - \overline{\Delta} + \Delta > 0 \quad \Leftrightarrow \quad \hat{x} > \bar{x}.$$

Hence, type $s_i < \bar{x}$ prefers 1A to 2A to 2B, type $s_i \in (\bar{x}, \hat{x})$ prefers 2B to 1A to 2A, and type $s_i \in \left(\hat{x}, \frac{1}{2}\right)$ prefers 2B to 2A to 1A. It follows that when $\Delta < \underline{\Delta}$, the optimal course of action is given by 1A if $s_i < \bar{x}$ and by 2B if $s_i > \bar{x}$. $\square$