Efficiency of Evolutionary Stability in Games of Common Interest with Pre-play Communication†

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Proposed Running Head: Evolutionarily Stable Set

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Abstract

This note studies the role of pre-play communication in two-person symmetric games of common interest when the message space is continuous or when communication is over (potentially) infinitely many rounds. It is shown that an evolutionarily stable set (Thomas (1985a,b) and Balkenborg and Schlag (1995)) is efficient and that the set of all efficient strategies forms an evolutionarily stable set.

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1. Introduction

In games of common interest, our basic intuition suggests that pre-play communication between players leads to an efficient outcome. Justifying this basic intuition in a purely non-cooperative paradigm, however, has turned out to be a major challenge for game theorists. The difficulty lies in the fact that there always exists an equilibrium in which communication has no significance and players take an inefficient action regardless of the outcome of the communication process. Although it is known that evolutionary stability eliminates some unwanted equilibria in cheap-talk games of common interest, this and other static evolutionary criteria alone do not predict efficiency in the conventional framework.[1] Consequently, much effort has been focused on constructing evolutionary dynamics that would have the efficient outcome as a stable state. Among others, Kim and Sobel (1995) and Matsui (1991) obtain positive results.

This note shows that when the message space used in pre-play communication is continuous, or when communication is over infinitely many rounds rather than one-shot, efficiency is implied by a set-wise extension of evolutionary stability known as an evolutionary stable set (Thomas (1985a,b)). In short, an evolutionarily stable set is a collection of strategies that satisfy the requirements of evolutionary stability against the strategies outside this set. Note that a strategy in standard cheap-talk games is the (possibly randomized) choice of a message as well as the (randomized) choice of an action based on the message profile. We say that a strategy in cheap-talk games of common interest is *efficient* if it induces the efficient outcome with probability one. When the message space is *finite,* it is known that (i) any strategy that does not use some of the messages belongs to an evolutionarily stable set if and only if it is efficient, and that (ii) there exists an inefficient strategy that uses (randomizes over) every message and forms an evolutionarily stable set by itself.[2] The existence of inefficient evolutionarily stable sets in models with a finite message space depends crucially on the fact that there exist strategies that send every message with positive probability. Our simple key observation is that when the message space is continuous, there exists *no*

[1]Note, however, that some efficiency results are obtained for a certain class of games. Wärneryd (1991) obtains efficiency of neutrally stable strategies in 2×2 coordination games with pre-play communication when only pure strategies are allowed. See also Wärneryd (1993).

[2]See Schlag (1993) and Weibull (1995, Section 2.6).

strategy that sends every message with positive probability. This observation leads us to the generalization of observation (i) above in our continuous framework. Moreover, the set of efficient strategies forms an evolutionarily stable set. Combined with the above result, this set is hence the *maximal* evolutionarily stable set.[3]

A few other papers study evolutionary stability in games with pre-play communication using a large message space. With a large but finite message space, Blume *et al.* (1993) establish the efficiency of evolutionarily stable sets in games of common interest when only one player talks and the other player responds by taking a payoff-relevant action. Although our propositions (and many of our arguments) resemble those in Blume *et al.* (1993), their conclusions depend critically on the assumption that communication and payoff-relevant actions are both one-sided. The logic behind their efficiency results does not extend to our two-sided framework. In fact, Schlag (1993) has shown that in 3×3 pure coordination games (with two-sided communication), an evolutionarily stable set can be highly inefficient even in the limit as the number of messages approaches infinity.[4] Banerjee and Weibull (1996) show that neutrally stable strategies can be inefficient in 2×2 (two-sided) cheap-talk coordination games with a countably infinite message space. It should be noted, however, that the requirement of an evolutionarily stable set is significantly stronger than that of neutral stability. Whether or not an evolutionarily stable set is efficient with a countably infinite message space remains an open question.

The efficiency results from the model with a continuous message space readily extend to the model with infinitely many rounds of communication. In this model, although players have a finite set of messages in each round, they may exchange messages (potentially) infinitely many times. In many situations, such a communication process may be considered more plausible than the standard one-shot formulation. It is again shown that any evolutionarily stable set contains only efficient strategies, and that the set of efficient strategies forms an evolutionarily stable set.

The organization of the paper is as follows: The next section formulates the basic model

[3]This may as well be the unique evolutionarily stable set. This, however, has not been confirmed.

[4]See Section 3. Schlag (1993) also shows that in 2×2 coordination games, an evolutionarily stable set is asymptotically efficient as the number of messages goes to infinity.

with a continuous message space. This model is analyzed in Section 3. Section 4 studies an alternative definition of an evolutionarily stable set due to Balkenborg and Schlag (1995). The model with infinitely many rounds of communication is studied in Section 5.

2. Model

The base game is a symmetric two-person game $G = (A, g)$, where A is an action set and $g : A^2 \to \mathbf{R}$ is a payoff function of each player. In what follows, we use the convention that the first argument is the action (message, strategy, *etc*) of the player in question and the second argument is that of the opponent. For example, $g(a, b)$ is the payoff to the player playing action a against the opponent playing action b. The action set A is a finite or bounded subset of $\mathbf{R}$. The game G is a *game of common interest* in the sense that there exists $a^* \in A$ such that for every $(a, b) \in A^2$, either (i) $g(a, b) = g(b, a) = g(a^*, a^*)$, or (ii) $g(a^*, a^*) > \max\{g(a, b), g(b, a)\}$. Namely, under any action profile, either both players receive the maximal payoff $g(a^*, a^*)$, or both receive strictly less than $g(a^*, a^*)$.[5] Prior to playing G, players can engage in costless communication about what action to choose in G. The message space is given by the unit interval $I = [0, 1]$ with the Borel σ-algebra, and two players simultaneously announce a number from I. After observing a pair of numbers, the players proceed to play G. Let C be the set of measurable functions from I^2 to A. Each player's pure strategy is a pair $s = (m, c)$, where $m \in I$ is the message and $c \in C$ chooses the action in G based on the realized message profile. Let $S = I \times C$ denote the set of pure strategies. Given a pure strategy profile $(s, t) \in S^2$, player i's payoff function $\pi : S^2 \to \mathbf{R}$ can be defined as

$$\pi(s, t) = g\big(c(m, n), d(n, m)\big),$$

where $s = (m, c)$ and $t = (n, d)$. Since g is bounded, so is π. Let $\mathcal{S}$ be a σ-algebra on S such that π is measurable with respect to the product σ-algebra $\mathcal{S}^2$.[6] A mixed strategy x is a probability distribution over $(S, \mathcal{S})$. Let X denote the set of mixed strategies.

A player's payoff function $\pi : X^2 \to \mathbf{R}$ can be extended to the set of mixed strategy

[5] The case where only one player receives $g(a^*, a^*)$ under some action profile will be discussed in Section 4.

[6] Note that in general π may fail to be continuous even if g is.

profiles in the usual manner:

$$\pi(x,y) = \int_{(s,t)\in S^2} \pi(s,t)\, dx(s)\, dy(t) \quad \text{for } (x,y) \in X^2.$$

The set X^2 is endowed with the topology of strong convergence of measures: $(x_n, y_n) \to (x,y)$ if and only if $\int_{S^2} f(s,t)\, dx_n(s)\, dy_n(t) \to \int_{S^2} f(s,t)\, dx(s)\, dy(t)$ for every bounded measurable function $f : S^2 \to \mathbf{R}$. The expected payoff function π is hence continuous in this topology.[7]

A symmetric strategy profile $(x,x) \in X^2$ is a *Nash equilibrium* if $\pi(x,x) \geq \pi(y,x)$ for any $y \in X$. Let $E \subset X$ denote the set of symmetric Nash equilibrium strategies, *i.e.*, $E = \{x \in X : (x,x) \text{ is a Nash equilibrium}\}$.

3. Analysis of an Evolutionarily Stable Set

The standard static criterion in evolutionary theory is that of evolutionary stability by Maynard Smith and Price (1973) as defined below:

DEFINITION: An equilibrium strategy $x \in E$ is *evolutionarily stable* if for any $y \in X$ with $\pi(y,x) = \pi(x,x)$,

(i) $\pi(x,y) \geq \pi(y,y)$, and

(ii) $\pi(x,y) = \pi(y,y)$ implies $y = x$.[8]

It turns out that in games with pre-play communication, the requirement of evolutionary stability is too strong (whether the message space is continuous or not). Consider a pure strategy s that sends signal 1 and plays the efficient action a^* regardless of the realized message profile. Although s induces an efficient outcome when matched against itself, it is not evolutionarily stable. To see this, suppose that t is a strategy which sends 1, plays a^* when the realized message profile is $(1,1)$, but plays some other action a otherwise. Since t induces exactly the same outcome when matched against s or t, Condition (ii) of evolutionary stability is violated.

Given that the problem is the existence of multiple strategies that differ from one another only "off the path," one possible solution is to consider the entire *set* of strategies that satisfy the conditions of evolutionary stability against any strategy outside this set. Thomas (1985a,b) proposes the following set-wise extension of evolutionary stability:

[7]See the discussion at the beginning of Section 4.

[8]$x \in E$ is *neutrally stable* if only (i) is imposed.

DEFINITION: A subset W of the set E of equilibrium strategies is an *evolutionarily stable set* if it is non-empty and closed, and for any $x \in W$, there exists an open neighborhood $U_x \subset X$ of x such that for any $y \in U_x$ with $\pi(y,x) = \pi(x,x)$,

(i) $\pi(x,y) \geq \pi(y,y)$, and

(ii) $\pi(x,y) = \pi(y,y)$ implies $y \in W$.

The following lemma provides basic characterizations of an evolutionarily stable set.[9]

LEMMA 1. *Suppose that W is an evolutionarily stable set. Then for any $x \in W$ and $y \in X$ with $\pi(y,x) = \pi(x,x)$,*

(i) $\pi(x,y) \geq \pi(y,y)$, and

(ii) $\pi(x,y) = \pi(y,y)$ implies $(1-\alpha)x + \alpha y \in W$ for any $\alpha \in [0,1]$.

PROOF: Let W be an evolutionarily stable set and take any $x \in W$ and $y \in X$ with $\pi(y,x) = \pi(x,x)$. For any $\alpha \in [0,1]$, let $z_\alpha = (1-\alpha)x + \alpha y \in X$ be the convex combination of x and y. Take an open neighborhood U_x of x as in the definition. There exists $\delta_{x,y} > 0$ such that for any $\epsilon \in (0, \delta_{x,y})$, $z_\epsilon \in U_x$. Take any such ϵ. Since $\pi(z_\epsilon, x) = (1-\epsilon)\pi(x,x) + \epsilon\pi(y,x) = \pi(x,x)$, we have by definition $\pi(x, z_\epsilon) \geq \pi(z_\epsilon, z_\epsilon)$. Since $\pi(x, z_\epsilon) = (1-\epsilon)\pi(x,x) + \epsilon\pi(x,y)$, and

$$\pi(z_\epsilon, z_\epsilon) = (1-\epsilon)\pi(z_\epsilon, x) + \epsilon\pi(z_\epsilon, y) = (1-\epsilon)\pi(x,x) + \epsilon\{(1-\epsilon)\pi(x,y) + \epsilon\pi(y,y)\},$$

$\pi(x, z_\epsilon) \geq \pi(z_\epsilon, z_\epsilon)$ is equivalent to $\pi(x,y) \geq \pi(y,y)$. This proves (i). Moreover, if $\pi(x,y) = \pi(y,y)$, then $\pi(x, z_\epsilon) = \pi(z_\epsilon, z_\epsilon)$ so that

$$z_\epsilon = (1-\epsilon)x + \epsilon y \in W \text{ for any } \epsilon \in (0, \delta_{x,y}). \tag{1}$$

For (ii), we first show that $y \in W$ if $\pi(x,y) = \pi(y,y)$. Note then that for any $\alpha \in [0,1]$, $\pi(z_\alpha, y) = (1-\alpha)\pi(x,y) + \alpha\pi(y,y) = \pi(y,y)$, and that

$$\begin{aligned}\pi(z_\alpha, z_\alpha) &= (1-\alpha)\pi(z_\alpha, x) + \alpha\pi(z_\alpha, y) \\ &= (1-\alpha)\{(1-\alpha)\pi(x,x) + \alpha\pi(y,x)\} + (1-\alpha)\{(1-\alpha)\pi(x,y) + \alpha\pi(y,y)\} \\ &= (1-\alpha)\pi(y,x) + (1-\alpha)\pi(y,y) \\ &= \pi(y, z_\alpha).\end{aligned}$$

[9] Essentially the same observation can be found in Balkenborg and Schlag (1995).

Let $\bar{\alpha}$ be the supremum of α's for which $z_\alpha \in W$: $\bar{\alpha} = \sup\{\alpha \in [0,1] : z_\alpha \in W\}$. Since W is closed, $\bar{z} = z_{\bar{\alpha}} \in W$. Suppose $\bar{\alpha} < 1$. Since $\pi(\bar{z}, y) = \pi(y, y)$ and $\pi(\bar{z}, \bar{z}) = \pi(y, \bar{z})$, we conclude from (1) (with $\bar{z}$ replacing x) that there exists $\delta_{\bar{z},y} > 0$ such that $(1-\epsilon)\bar{z} + \epsilon y \in W$ for any $\epsilon \in (0, \delta_{\bar{z},y})$. However, since

$$(1-\epsilon)\bar{z} + \epsilon y = (1-\epsilon)\{(1-\bar{\alpha})x + \bar{\alpha}y\} + \epsilon y = (1-\bar{\alpha})(1-\epsilon)\,x + \{\epsilon + (1-\epsilon)\bar{\alpha}\}\,y,$$

and $\epsilon + \bar{\alpha}(1-\epsilon) > \bar{\alpha}$ when $\bar{\alpha} < 1$, this is a contradiction to the fact that $\bar{\alpha}$ is the supremum. Therefore, $\bar{\alpha} = 1$ and hence $y \in W$.

Finally, since $\pi(z_\alpha, x) = \pi(x, x)$ and $\pi(x, z_\alpha) = \pi(z_\alpha, z_\alpha)$ for any $\alpha \in [0,1]$ if $\pi(y, x) = \pi(x, x)$ and $\pi(x, y) = \pi(y, y)$, repeating the same argument as above with z_α in place of y proves $z_\alpha = (1-\alpha)x + \alpha y \in W$. //

It can be seen from Lemma 1 that if x is an element of an evolutionarily stable set W, then any strategy y for which either $\pi(y, x) < \pi(x, x)$ or $\pi(y, y) < \pi(x, y)$ cannot invade the population playing x. In particular, no $y \notin W$ can invade x since then $\pi(y, y) < \pi(x, y)$. If $\pi(y, x) = \pi(x, x)$ and $\pi(x, y) = \pi(y, y)$, then $y \in W$ and it may survive as a drift. In this case, however, any convex combination of x and y is also an element of W.

Given a strategy $x \in X$, let λ_x denote the marginal distribution of x on I and μ_x denote the marginal distribution of x on A conditional on the message profile (m, n). Namely,

$$\lambda_x(B) = x(B \times C) \quad \text{for every Borel set } B \subset I \text{, and}$$
$$\mu_x(B \mid m, n) = x\big(I \times \{c \in C : c(m, n) \in B\}\big) \quad \text{for every Borel set } B \subset A.$$

Namely, $\lambda_x(B)$ is the probability that x chooses a message from B, and $\mu_x(B \mid m, n)$ is the probability that x chooses an action from B when the realized message profile is (m, n). Let $Y = \{x \in X : \pi(x, x) = g(a^*, a^*)\}$ be the set of strategies that induce the efficient outcome with probability one. The following proposition states that if W is an evolutionarily stable set, then any strategy in W must be efficient.

Proposition 1. *If W is an evolutionarily stable set, then $W \subset Y$.*

Proof: We derive a contradiction by supposing that there exists $x \in W$ such that $x \notin Y$. Let F_x be the cumulative distribution function over I corresponding to λ_x: $F_x(m) =$

$\lambda_x([0,m])$ for all $m \in I$. Since F_x has only countably many points of discontinuity, there exists $m^* \in I$ such that F_x is continuous at m^*, or, $\lambda_x(\{m^*\}) = 0$.

Let $y \in X$ be the strategy such that $\lambda_y = \lambda_x$, $\mu_y(\{a^*\} \mid m, m^*) = 1$ for any $m \in I$, and $\mu_y(B \mid m,n) = \mu_x(B \mid m,n)$ for any $B \subset A$, $m \in I$ and $n \neq m^*$. Namely, y sends a message according to the same probability distribution as x, plays an action according to the same probability distribution as x except for the case when the opponent's message is m^*, in which case it plays a^* for sure.

Since $\pi(y,x) = \pi(x,x)$ and $\pi(x,y) = \pi(y,y)$, it follows from Lemma 1 that $y \in W$. Let $w \in X$ be a strategy such that $\lambda_w(\{m^*\}) = 1$ and $\mu_w(\{a^*\} \mid m,n) = 1$ for any $(m,n) \in I^2$. Clearly, $\pi(w,y) = g(a^*,a^*) > \pi(y,y)$. It follows that $y \notin E$, a contradiction. //

The following proposition states that the set of efficient strategies forms an evolutionarily stable set. Combined with Proposition 1, it then implies that Y is the *maximal* evolutionarily stable set.

PROPOSITION 2. *Y is an evolutionarily stable set.*

PROOF: The set Y is clearly non-empty and $Y \subset E$. It is also closed: For any sequence of strategies (x_n) in Y converging to x, we have $x \in Y$ since π is continuous so that

$$\pi(x,x) = \pi\big(\lim_{n\to\infty} (x_n, x_n)\big) = \lim_{n\to\infty} \pi(x_n, x_n) = 1.$$

Take any $x \in Y$ and any open neighborhood $U_x \subset X$ of x. Take any $y \in U_x$ such that $\pi(y,x) = \pi(x,x)$. Since $\pi(y,x) = g(a^*,a^*)$, it must be the case that $\pi(x,y) = g(a^*,a^*)$ by our assumption about the base game G. We thus have $\pi(x,y) \geq \pi(y,y)$. If in fact $\pi(y,y) = \pi(x,x)$, then $y \in Y$. This completes the proof. //

In our definition of games of common interest, either both players receive the same maximal payoff $g(a^*,a^*)$, or both receive strictly less than $g(a^*,a^*)$. Suppose that we relax this assumption so that under some action profile, one player receives $g(a^*,a^*)$ while the other receives strictly less. In this case, Proposition 2 may no longer be true. To see this, consider the base game G in Figure 1, where (a,a) is the unique efficient profile. Note also that a is a weakly dominated action. Since $g(a,a) = g(b,a) > g(a,b)$, it is not a game of common interest as defined in this paper.

\	a	b
a	2,2	0,2
b	2,0	1,1

Figure 1

It can be readily verified that *no* strategy $x \in X$ such that (x,x) induces a with probability one can be an element of an evolutionarily stable set. In view of Proposition 1,[10] we conclude that there exists no evolutionarily stable set.

Schlag (1993) shows that an evolutionarily stable set can be very inefficient with a large but finite message space. This implies that the result in the continuous case is not the limit of that in the finite case as the number of messages goes to infinity. Schlag's (1993) example of asymptotic inefficiency is reproduced below to illustrate this point.

\	a	a'	a''
a	3,3	0,0	0,0
a'	0,0	2,2	0,0
a''	0,0	0,0	1,1

Figure 2

Consider the base game G in Figure 2 and let the message space be given by $M = \{1, \ldots, n\}$. For each $m \in M$, a pure strategy s_m sends message m and plays a'' if the opponent's message is also m but plays a' otherwise. Let x be the mixed strategy that mixes $s_1, \ldots, s_n$ with equal probability. It can be readily verified that x is an inefficient equilibrium strategy with $\pi(x,x) = 2 - \frac{1}{n}$. Furthermore, $s_1, \ldots, s_n$ are the only pure best responses to x, and if $y \neq x$ is a convex combination of $s_1, \ldots, s_n$, then $\pi(y,y) < \pi(x,y) = \pi(y,x) = 2 - \frac{1}{n}$. It follows that for any n, $W = \{x\}$ is an evolutionarily stable set whose efficiency is bounded above by 2.

4. Alternative Definition of an Evolutionarily Stable Set

It should be noted that the choice of a topology for X^2 is critical for the conclusion of Proposition 2 in Section 3.[11] We have chosen the topology of strong convergence of measures

[10] Proposition 1 holds for this class of games.

[11] Proposition 1, on the other hand, is true under weaker topologies.

for X^2. This guarantees the continuity of the expected payoff function π and hence the closedness of set Y. Suppose instead that X^2 is endowed with the topology of weak convergence of measures: $(x_n, y_n) \to (x, y)$ if and only if $\int_{S^2} f(s,t)\, dx_n(s)\, dy_n(t) \to \int_{S^2} f(s,t)\, dx(s)\, dy(t)$ for every bounded *continuous* function $f : S^2 \to \mathbf{R}$. It can be seen that Y is not closed under this topology: Consider the following sequence of strategies (x_n): x_n sends $0.5(1 + 1/n)$ for sure, and plays a^* for sure if the opponent's message is strictly greater than 0.5 and some other action a otherwise. Let x be the strategy which sends 0.5 for sure and plays an action the same way as x_n. Clearly, $x_n \in Y$ and $x \notin Y$. It can be readily verified, however, that (x_n, x_n) converges to (x, x) in the topology of weak convergence while it does not in the topology of strong convergence. The set Y is hence not closed in this weaker topology.

The problem arises from the requirement that an evolutionarily stable set be closed in the definition by Thomas (1985a,b). This requirement is part of his definition since it compares $x \in W$ only with strategies $y \in X$ in its neighborhood. As Thomas (1985b) notes, however, introduction of a neighborhood in his definition of an evolutionarily stable set is motivated by its adaptability to environments with non-linear payoff functions. Since our focus is on the standard bilinear payoff functions, a more straightforward definition would simply compare $x \in W$ with every strategy $y \in X$ as in the definition of evolutionary stability. This observation leads us to the following definition of an evolutionarily stable set, which is first proposed by Balkenborg and Schlag (1995):

DEFINITION: A subset of equilibrium strategies $W \subset E$ is an *evolutionarily stable set* if it is non-empty, and for any $x \in W$ and $y \in X$ such that $\pi(y, x) = \pi(x, x)$, we have

(i) $\pi(x, y) \geq \pi(y, y)$, and

(ii) $\pi(x, y) = \pi(y, y)$ implies $y \in W$.

Note that this definition does not require that W be closed. In essence, it captures those properties of the original evolutionarily stable set that are described by Lemma 1.[12] With this alternative definition of an evolutionarily stable set, it can be shown that the conclusions of Section 3 are true when X^2 is endowed with the topology of weak convergence of measures. As before, let Y be the set of efficient strategies.

[12]Note that "$y \in W$" in Condition (ii) of the definition is interchangeable with "$(1-\alpha)x + \alpha y \in W$ for every $\alpha \in [0,1]$."

PROPOSITION 3. *If W is an evolutionarily stable set, then $W \subset Y$. Moreover, Y is an evolutionarily stable set.*

PROOF: The first claim is based on the same logic as that of the proof of Proposition 1. For the second claim, take any $x \in Y$ and $y \in X$ such that $\pi(y,x) = \pi(x,x)$. Since $\pi(y,x) = g(a^*,a^*)$, it must be the case that $\pi(x,y) = g(a^*,a^*)$ by our assumption about the base-game G. Therefore, $\pi(x,y) \geq \pi(x,y)$. If the inequality holds with equality, then $\pi(y,y) = g(a^*,a^*)$, and hence $y \in Y$. //

5. Pre-play Communication over Infinitely Many Rounds

The results with a continuous message space in Sections 3 and 4 readily extend to the case where the message space in each round is finite, but communication may extend over infinitely many rounds.[13] Let $M = \{0,1,\ldots,k\}$ be the finite message space of each player in each round. Message 0 may be identified as no message. This interpretation allows for the possibility that players choose to stop communication after some point. An *n-length communication history* is a string of pairs of messages over n rounds of communication. Let $H^n = M^{2n}$ be the set of n-length communication histories for $n = 0,1,\ldots,\infty$ ($H^0 = \{\phi\}$), and let $H = \bigcup_{n=0}^{\infty} H^n$ be the set of finite communication histories. A player's pure *communication strategy* σ is a mapping from H to M. Let Σ be the set of pure communication strategies. We use $h_{\sigma,\tau} \in H^\infty$ to denote the infinite communication history induced by the communication strategy profile $(\sigma,\tau) \in \Sigma^2$. When C is the set of measurable functions from H^∞ (with the product σ-algebra) to A, a player's pure strategy $s = (\sigma,c)$ is an element of $S = \Sigma \times C$, where $c \in C$ chooses the action given an infinite communication history. Given a pure strategy profile $(s,t) \in S^2$, a player's payoff function π is defined by

$$\pi(s,t) = g\big(c(h_{\sigma,\tau}),\, d(h_{\sigma,\tau})\big),$$

where $s = (\sigma,c)$ and $t = (\tau,d)$. A mixed strategy $x \in X$ is a probability distribution on $(S,\mathcal{S})$, where $\mathcal{S}$ is an appropriate σ-algebra. Given a mixed strategy profile $(x,y) \in X^2$, the expected payoff $\pi(x,y)$ is defined in the same manner as before. The definitions of an

[13]Note that such a communication process need not take an infinite amount of time. For example, if the nth exchange of messages takes place at time $1-0.5^n$, then the entire process takes one unit of time.

equilibrium strategy and an evolutionarily stable set remain unchanged. For any strategy x, let λ_x be the marginal distribution of x on Σ, and $\mu_x(\cdot \mid h)$ be the marginal distribution of x on A conditional on the infinite communication history $h \in H^\infty$.

PROPOSITION 4. *In symmetric games of common interest with infinitely many rounds of pre-play communication, if W is an evolutionarily stable set (in either definition), then $W \subset Y$. Moreover, Y is an evolutionarily stable set.*

PROOF: Write each infinite communication history $h \in H^\infty$ as $h = (\alpha, \beta)$, where α and $\beta \in M^\infty$ are infinite strings of each player's messages. Given each infinite communication history $h = (\alpha, \beta)$, note that both α and β correspond to points in the interval $I = [0,1]$ when numbers in this interval are expressed in base $k+1$. Formally, there exists a surjective mapping $r : M^\infty \to I$ defined by $r(\alpha) = 0.\alpha_1\alpha_2 \cdots \alpha_n \cdots$ for each $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n, \ldots) \in M^\infty$, where the number is written in base $k+1$.[14] For example, if $k = 1$, then $r(\alpha) = 0.\alpha_1\alpha_2 \cdots \alpha_n \cdots$ ($\alpha_n = 0$ or 1) is a binary expansion of a number in the interval $[0,1]$.

Let $\nu_{x,y}$ be the marginal probability distribution of $\alpha \in M^\infty$ (an infinite string of messages by the player whose strategy is x) induced by the strategy profile $(x,y) \in X^2$. Let $F_{x,y}$ be the corresponding distribution function under the above identification of α and a number in $[0,1]$: $F_{x,y}(\eta) = \nu_{x,y}(r^{-1}([0,\eta]))$ for any $\eta \in [0,1]$.

Suppose that $x \in W$ and $x \notin Y$. Since $F_{x,x}$ has only countably many points of discontinuity, there exists $\eta^* \in I$ such that F is continuous at η^*, or, $\nu_{x,x}(r^{-1}(\{\eta^*\})) = 0$. Since r is surjective, we can take $\alpha^* \in r^{-1}(\{\eta^*\})$. Consider a strategy $y \in X$ such that $\lambda_y = \lambda_x$, $\mu_y(\{a^*\} \mid \alpha, \alpha^*) = 1$ for any α, and $\mu_y(\cdot \mid \alpha, \beta) = \mu_x(\cdot \mid \alpha, \beta)$ for any α and $\beta \neq \alpha^*$. Clearly, $\pi(y,x) = \pi(x,x)$ and $\pi(x,y) = \pi(y,y)$. We can hence conclude that $y \in W$ under either definition of an evolutionarily stable set. Let $\sigma^* \in \Sigma$ be the communication strategy that sends α_n^* in round n regardless of communication histories up to round $n-1$. Consider a strategy $w \in X$ such that $\lambda_w(\{\sigma^*\}) = 1$, and $\mu_w(\{a^*\} \mid h) = 1$ for any $h \in H^\infty$. We have a contradiction that $y \notin E$ since $\pi(w,y) = g(a^*,a^*) > \pi(y,y)$. The rest of the proof is the same as that of Proposition 2 or 3, and hence is omitted. //

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[14]Note that r is not injective since $0.199\cdots$ and 0.2, for example, are the same number.

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